MAT 121-G11 Test 2 – Fall, 2013 (5 pts) Name________________

100 Points Covers Chapter 2 NO GRAPHING CALCULATORS!!!

1. (10 pts) Is the relation $f = \{(3,-7),(4,-1),(2,5),(9,-1)\}$ a function? Explain in words.

Yes. To each x, there is exactly one y.

2. (5 pts) What's the domain of f?

$\mathcal{D} = \{3, 4, 2, 9\}$

3. (5 pts) What's the range of f?

$\mathcal{R} = \{-7, -1, 5\}$

4. Let $f(x) = \dfrac{x+7}{x-2}$ and $g(x) = \sqrt{x+1}$.

a. (5 pts) What is the domain of f ?

$\{x \mid x \neq 2\}$

b. (5 pts) What is the domain of g ?

$\{x \mid x \geq -1\}$

c. Determine the following functions. You don't need to simplify. In fact, I recommend you do not.

i) (5 pts) $f+g = \dfrac{x+7}{x-2} + \sqrt{x+1}$

ii) (5 pts) $f \circ g = \dfrac{\sqrt{x+1} + 7}{\sqrt{x+1} - 2}$

d. (5 pts) What is the domain of $f+g$? $\{x \mid x \neq 2 \text{ and } x \geq -1\}$

INTERVAL NOTATION $= [-1, 2) \cup (2, \infty)$

e. (5 pts) What is the domain of $f \circ g$? $= \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

INTERVAL NOTATION $= \{x \mid x \geq -1 \text{ and } \sqrt{x+1} \neq 2\}$

$= \{x \mid x \geq -1 \text{ and } x \neq 3\}$

$= [-1, 3) \cup (3, \infty)$

$\sqrt{x+1} = 2$

$x+1 = 4$

$x = 3$

5. (5 pts) Simplify the difference quotient, $\dfrac{f(x+h)-f(x)}{h}$, for $f(x)=x^2-5x$.

$$\frac{(x+h)^2-5(x+h)-(x^2-5x)}{h} = \frac{x^2+2xh+h^2-5x-5h-x^2+5x}{h}$$

$$= \frac{2xh+h^2-5h}{h} = \boxed{2x+h-5}$$

Bonus Pass to the limit, as $h \to 0$, on your answer to the above, so you can show me some calculus.

$$\xrightarrow{h \to 0} \boxed{2x-5}$$

6. (5 pts) Explain to me why the equation $x^2+y^2=49$ does *not* define y as a function of x.

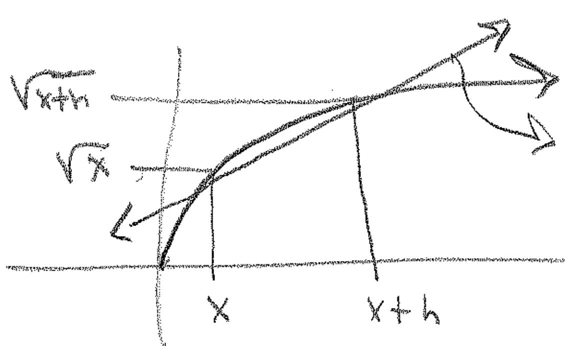
$y^2 = 49 - x^2$

$y = \pm\sqrt{49-x^2}$

Gives 2 y-values for 1 x-value.

$x=0 \Rightarrow y=\pm 7 \Rightarrow (0,7), (0,-7) \in$ Relation.

7. (5 pts) Draw me a picture showing what the difference quotient represents for the function $f(x)=\sqrt{x}$.



This line has slope

$$m = \frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h}$$

Bonus Simplify the difference quotient for $f(x)=\sqrt{x}$.

$$\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$=\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

8. (10 pts) Answer one of the following:

   a. Show that $f(x)=\frac{2}{3}x-7$ is one-to-one, algebraically.

   b. If $f(x)=x^2-6x+7$, for $x\geq 3$, what is $f^{-1}(x)$?.

(a) $\frac{2}{3}x_1-7=\frac{2}{3}x_2-7$

$\frac{2}{3}x_1=\frac{2}{3}x_2$

$x_1=x_2$

(b) $y^2\ 6y+7=x$

$y^2\ 6y+3^2=x-7+9$

$(y-3)^2=x+2$

$y-3=\pm\sqrt{x+2}\longrightarrow+\sqrt{x+2}$

$y=\boxed{3+\sqrt{x+2}=f^{-1}(x)}$

9. (10 pts) Show that $f(x)=\dfrac{x+3}{x-1}$ is its own inverse. In other words, show that in this example, f and f^{-1} are the same, exact function! There are two ways to accomplish this:

   1. By finding f^{-1}, directly.
   2. By the definition of f^{-1}.

(1) $\dfrac{y+3}{y-1} = x$

$y+3 = xy - x$

$y - xy = -x-3$

$y(1-x) = -x-3$

$y = \dfrac{-x-3}{1-x} = \dfrac{x+3}{x-1}$

$= f^{-1}(x) = f(x)$!

$(f\circ f^{-1})(x) =$

$$\frac{\frac{x+3}{x-1}+3}{\frac{x+3}{x-1}-1} = \frac{\frac{x+3+3(x-1)}{x-1}}{\frac{x+3-(x-1)}{x-1}}$$

$$= \frac{\frac{x+3+3x-3}{x-1}}{\frac{x+3-x+1}{x-1}}$$

$$= \frac{\frac{4x}{x-1}}{\frac{4}{x-1}}$$

$$= \frac{4x}{x-1}\cdot\frac{x-1}{4} = x \checkmark$$

10. (10 pts) Suppose y varies jointly with m_1 and m_2, and inversely with the square of r. Write an equation describing this situation.

$y = k\dfrac{m_1 m_2}{r^2}$