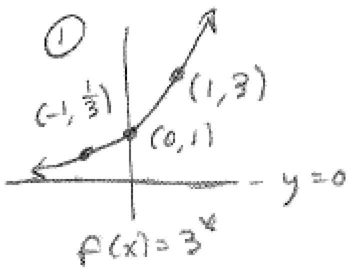
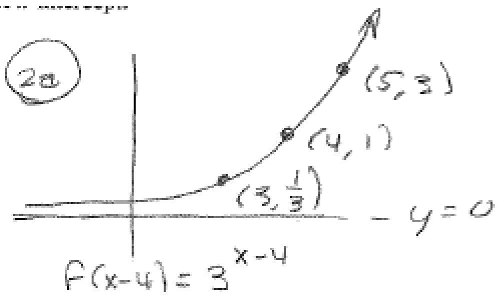
FINAL WED 7:10am

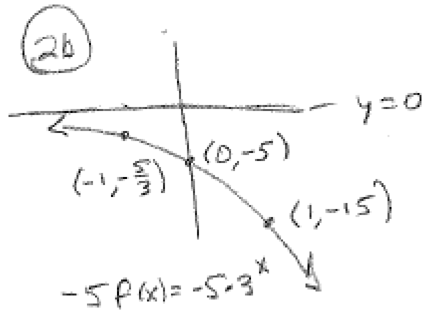
①



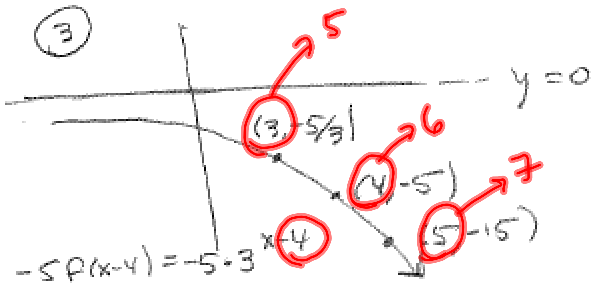
②a

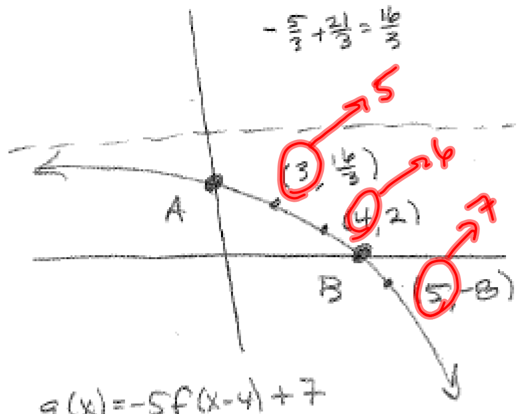


②b



③





$g(x) = -5f(x-4)+7$

$= -5\cdot 3^{x-4}+7$

$= g(x)$

Fix on next pg.

A: $g(0) = -5\cdot 3^{-4}+7$ 6

$= -\frac{5}{81} + \frac{567}{81}$ 729 $\frac{1503}{729}$

$= \frac{562}{81} \approx \boxed{A = (0, \frac{562}{81})}$

$\approx (0, 6.93827)$

6.938271605

B: $-5\cdot 3^{x-4}+7=0$

$-5\cdot 3^{x-4} = -7$

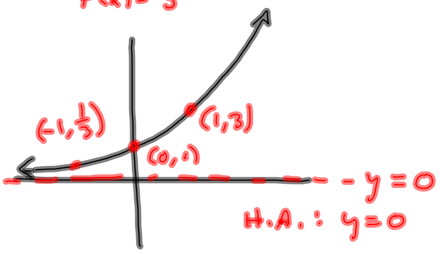
$3^{x-4} = \frac{7}{5}$

$x-4 = \log_3(7/5)$

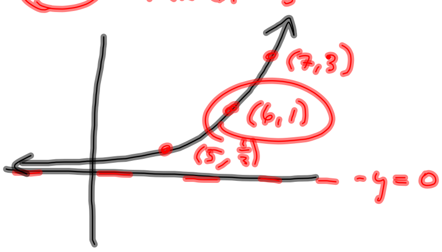
$x = 4+\log_3(\frac{7}{5}) \approx 4 + \frac{\ln(7/5)}{\ln(3)} \approx 4.306270278$

$B = (4+\frac{\ln(7/5)}{\ln(3)}, 0) \approx \boxed{(4.30627, 0) \approx B}$

$f(x) = 3^x$

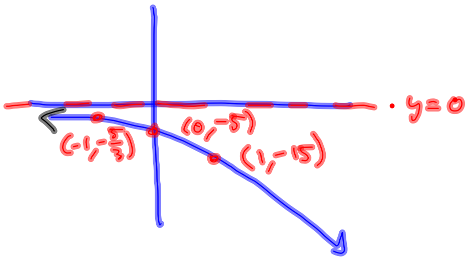


(2a) $f(x-6) = 3^{x-6}$

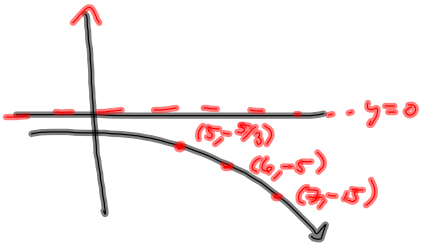


(2b) $-5f(x) = -5\cdot 3^x$

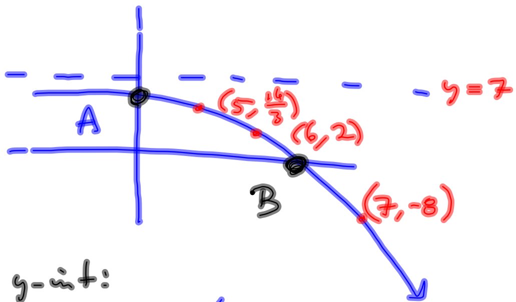
Kel: pissed me off.



(3) $-5f(x-6) = -5\cdot 3^{x-6}$



(4) $-5\cdot 3^{x-6}+7 = -5f(x-6)+7 = g(x)$



$-\frac{5}{3}+\frac{7}{1}\cdot\frac{3}{3} = \frac{16}{3}$

y-int:

$g(0) = -5\cdot 3^{0-6}+7$

$= \frac{-5}{729} + \frac{7(729)}{729}$

$= \frac{-5+5103}{729} = \frac{5098}{729}$

$\boxed{A = (0, \frac{5098}{729})} \approx (0, 6.993141289)$

x-int:

$g(x) = 0$

$-5\cdot 3^{x-6}+7 = 0$

$-5\cdot 3^{x-6} = -7$

$3^{x-6} = \frac{7}{5}$

$\log_3(3^{x-6}) = \log_3(\frac{7}{5})$

$x-6 = \log_3(\frac{7}{5})$

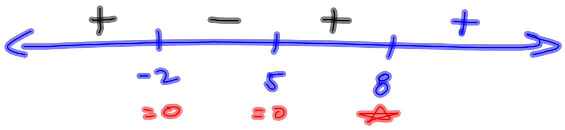
$x = \log_3(\frac{7}{5})+6$

$B = (\log_3(\frac{7}{5})+6, 0)$

$\approx (6.30627, 0)$

$\frac{\ln(7/5)}{\ln(3)} + 6$

3 $\quad \dfrac{(x-5)(x+2)^3}{(x-8)^2} \geq 0$



$(-\infty,-2] \cup [5,8) \cup (8,\infty)$

4

$5^{2y-5} - 3 = x$

$5^{2y-5} = x + 3$

$\log_5(5^{2y-5}) = \log_5(x+3)$

$* \quad 2y-5 = \log_5(x+3)$

$2y = \log_5(x+3) + 5$

$y = \dfrac{\log_5(x+3)+5}{2} = f^{-1}(x)$

$\log_5(5^{2y-5})$

$= (2y-5)\log_5(5)$

$= 2y-5$

5a $\quad 5 + 10 + 20 + 40 + \overset{80+160+}{\cdots} + 320$

$a = 5$

$r = \frac{10}{5} = \frac{20}{10} = \frac{40}{20} = 2$

$320 = ar^{n-1} = 5 \cdot 2^6$

$n-1 = 6$

$n = 7$

2 | 320
2 | 160
2 | 80
2 | 40
2 | 20
2 | 10
5

$\displaystyle\sum_{k=1}^{n} ar^{k-1} = a\left(\frac{1-r^n}{1-r}\right) = a\left(\frac{r^n-1}{r-1}\right)$

$= 5\left(\frac{1-2^7}{1-2}\right) = 5\left(\frac{-127}{-1}\right) = 635$

5b

$\displaystyle\lim_{n\to\infty}\left(\frac{5}{7}\right)^n = 0$

$\displaystyle\sum_{k=1}^{\infty} 3\cdot\left(\frac{5}{7}\right)^{k-1} = \lim_{n\to\infty}\sum_{k=1}^{n} 3\cdot\left(\frac{5}{7}\right)^{k-1}$

$\displaystyle= \lim_{n\to\infty} 3\left(\frac{1-\left(\frac{5}{7}\right)^n}{1-5/7}\right) = 3\left(\frac{1}{1-\frac{5}{7}}\right) = \frac{21}{2}$

$a\left(\frac{1}{1-r}\right)$

$$\log_2(x+14)+\log_2(x+18)=5$$

$$\log_2\big((x+14)(x+18)\big)=5$$

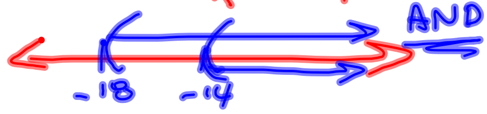
$$2^{\log_2\left((x+14)(x+18)\right)}=2^5$$

$\mathcal{D}$:

Need $x+14>0$ and $x+18>0$, i.e.,

$x+14>0$

$x>-14$



$$x^2+32x+252=32$$

$$x^2+32x+220=0$$

$\vdots$

$x=-10,-22 \Rightarrow$

$\boxed{x=-10}$ b/c $x=-22\notin\mathcal{D}$

$\log_2(-10+14)$
$+\log_2(-10+18)$
$=\log_2(4)+\log_2(8)$
$=2+3=5$ ✓

$A(t) = A_0 e^{kt}$

$\boxed{\tfrac{1}{2}\text{-life} = 5200 \text{ yrs.}}$

$A(5200) = \boxed{A_0 e^{5200k} = \tfrac{1}{2} A_0}$

$e^{5200k} = \frac{1}{2}$

$\ln_3(x)$ means nothing to me.

$\log_3(x)$ is what you mean.

$\ln(e^{5200k}) = \ln(\tfrac{1}{2})$

$5200k = \ln(1/2) = -\ln(2)$

$$\boxed{k = \frac{-\ln(2)}{5200}}$$

$\ln(x) = \log_e(x)$

$\log_e(e^x) = x$

65% is gone!

35% remains!

we have k from part a. Solve for t.

$A_0 e^{kt} = .35 A_0$

$e^{kt} = .35$

$\ln(e^{kt}) = \ln(.35)$

$kt = \ln(.35)$

$= \frac{5200 \ln(.35)}{\ln(1/2)}$

$$t = \frac{\ln(.35)}{k} = \frac{\ln(.35)}{\frac{-\ln(2)}{5200}} =$$

$$= \boxed{\frac{-5200 \ln(.35)}{\ln(2)}} \approx 7875.780499$$

≈ 7876 yrs old.