§ 4.4 #41

$6^x = 3^{x+1}$ $\qquad \mathcal{D} = \mathbb{R}$

$\ln(6^x) = \ln(3^{x+1})$

$x \ln 6 = (x+1)\ln 3$

$ax = b(x+1) \qquad \ln 6 = a, \ \ln 3 = b$

$ax = bx + b$

$ax - bx = b$

$x(a-b) = b$

$(a-b)x = b$

$x = \dfrac{b}{a-b} = \dfrac{\ln 3}{\ln 6 - \ln 3} = \dfrac{\ln 3}{\ln 2}$

≈ 1.5850

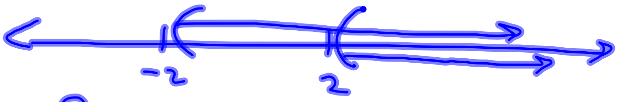
Teacher's confused about domain :

RULE : Do Domain of original problem.

(15) $\log_2(x+2)+\log_2(x-2)=5$

$\mathcal{D}$: Need $x+2>0$ and $x-2>0$

$x>-2$ and $x>2$



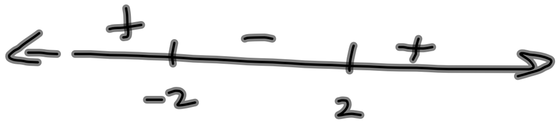
$\mathcal{D}=\{x \mid x>2\}=(2,\infty)$

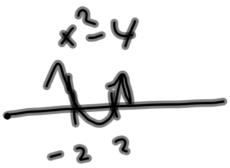
After next step:

$\log_2\left((x+2)(x-2)\right)=5$

Domain of this problem:

Need $(x+2)(x-2)>0$





This says $x<-2$

OR

$x>2$

Now we're dealing with the PRODUCT of $x+2$ & $x-2$

Different deal.

$\mathcal{D}=\{x \mid x<-2 \text{ OR } x>2\}$

$=(-\infty,-2)\cup(2,\infty)$

which is why I'm confused.

But we MUST roll with the $\mathcal{D}$ of original.

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$\log_3(x) = \log_3(2) - \log_3(x-2)$

$\log_3(x) = \log_3\left(\frac{2}{x-2}\right)$

$\mathcal{D}$: $x>0$ and $x>2$ $\Rightarrow x>2$

$x = \frac{2}{x-2}$

$\Rightarrow x(x-2) = 2$

$x^2 - 2x = 2$

$x^2 - 2x - 2 = 0$

$2 \lfloor 12$, $2 \lfloor 6$, 3

$a=1, b=-2, c=-2$

$b^2 - 4ac = (-2)^2 - 4(1)(-2)$

$= 4 + 8$

$= 12$

$\rightsquigarrow \sqrt{12} = \sqrt{2 \cdot 2 \cdot 3} = 2\sqrt{3}$

$x = \frac{2 \pm 2\sqrt{3}}{2(1)} = \frac{2(1 \pm \sqrt{3})}{2} = 1 \pm \sqrt{3}$

$$\boxed{\begin{aligned} x^2 - 2x + 1^2 &= 2 + 1 \\ (x-1)^2 &= 3 \\ x - 1 &= \pm\sqrt{3} \\ x &= 1 \pm \sqrt{3} \end{aligned}}$$

$\boxed{1+\sqrt{3}} \approx 2.7$ish

Final answer.

$\boxed{1 - \sqrt{3} \notin \mathcal{D}}$

$x \in \{1+\sqrt{3}\}$ is good

$x = \{1+\sqrt{3}\}$ isn't appropriate

$x = 1+\sqrt{3}$ is good.

5. Find the geometric sums:

a. (10 pts) $5 + 10 + 20 + 40 + \ldots\ldots + 320$

$a = 5$

$r = \frac{10}{5} = \frac{20}{10} = \frac{40}{20} = 2 =$ common ratio

$5 \cdot 2^6 = 320 = ar^{n-1} = 5 \cdot 2^{n-1}$

$6 = n - 1$

$7 = n$

$\sum_{k=1}^{n} ar^{k-1} = a\left(\frac{r^n - 1}{r - 1}\right) = 5\left(\frac{2^7 - 1}{2 - 1}\right)$

$= 5\left(\frac{128 - 1}{1}\right) = 5\,(127) = 635$

$2 \,|\, 320$
$2 \,|\, 160$
$2 \,|\, 80$
$2 \,|\, 40$
$2 \,|\, 20$
$2 \,|\, 10$
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b. (5 pts) $\sum_{n=1}^{\infty}\left(\frac{2}{5}\right)^{n-1}$ $= \left(\frac{2}{5}\right)^0 + \left(\frac{2}{5}\right) + \left(\frac{2}{5}\right)^2 + \cdots$

$= 1 + \frac{2}{5} + \frac{4}{25} + \cdots$

$a = 1$

$r = \frac{2}{5}$

Think of infinite sum (i.e., series) as the limit of a finite sum.

$\sum_{n=1}^{\infty} = \lim_{m\to\infty} \sum_{n=1}^{m}$

$\lim_{m\to\infty} \sum_{n=1}^{m} 1\cdot(2/5)^{n-1} = \lim_{m\to\infty} \boxed{a\left(\frac{r^m - 1}{r-1}\right)}$

$r^m \to 0$ if $r<1$

$0<r<1$

$= \lim_{m\to\infty}\left(\frac{(2/5)^m - 1}{\frac{2}{5}-1}\right)$

$= \frac{0-1}{\frac{2}{5}-1} = \frac{-1}{-\frac{3}{5}} = \frac{5}{3}$

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when $0<r<1$

$\sum_{k=1}^{\infty} ar^{k-1} = a\left(\frac{-1}{r-1}\right) = a\left(\frac{1}{1-r}\right)$

$a\left(\frac{1-r^n}{1-r}\right)$

Suppose the half-life of C-14 is 5800 years. (It isn't, quite, but just suppose…).

a. (10 pts) Derive the exponential decay model, $A(t) = A_0 e^{kt}$. The trick is to use the half-life to find the relative decay rate, k.

Exponential Growth/Decay: $A(t) = A_0 e^{kt}$

$A_0 e^{5800k} = \frac{1}{2} A_0$, since $A(0) = A_0 e^0 = A_0$

$e^{5800k} = \frac{1}{2}$ $\quad \ln x =$

$\ln(e^{5800k}) = \ln(\frac{1}{2})$ $\quad \ln(\frac{1}{2}) = \ln(2^{-1}) = -\ln(2)$

$5800k = \ln(\frac{1}{2}) = -\ln 2$

$$\boxed{k = -\frac{\ln 2}{5800}}$$

b. ~~(5 pts) How old~~ is a sample of charcoal from a prehistoric fire pit, if 80% of the C-14 has decayed (i.e., 20% is left.) ?

$A_0 e^{kt} = .2 A_0$

$e^{kt} = .2$

$\ln(e^{kt}) = \ln(.2)$

$kt = \ln(.2)$

$$\boxed{t = \frac{\ln(.2)}{k}} = \frac{\ln(.2)}{-\frac{\ln(2)}{5800}}$$

$$= \ln(.2) \cdot \frac{5800}{-\ln(2)} = \frac{5800 \ln(.2)}{-\ln(2)}$$

$= \quad \approx 13467.18295$

t	A	
5800	$\frac{1}{2}A_0$	.5
11600	$\frac{1}{4}A_0$	.25
17400	$\frac{1}{8}A_0$	.125

$\leftarrow .2$

$\log_e x$ & e^x are inverse functions

e^e

$e^0 = 1$ $5800k \ln e = 5800k \log_e e = 5800k \cdot 1$

x $e^1 = e^?$ $? = 1$

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