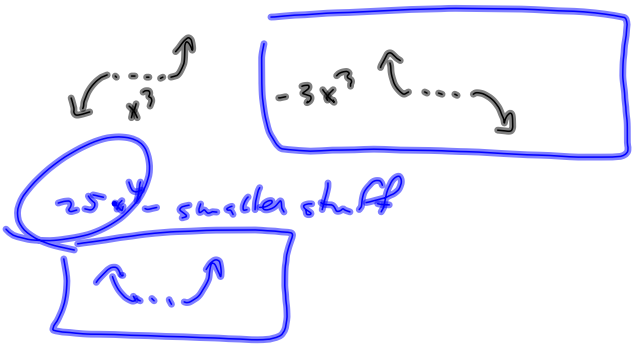
Take-Home

$-3x^3+7x^2$



$4x^5 - 12x^4 - 5x^3 + 21x^2 - 11x - 21$

1 2 3

3 or 1 positive zeros

$f(-x) = -4x^5 - 12x^4 + 5x^3 + 21x^2 + 11x - 21$

1 2

2 or 0 neg. zeros

p's: $\pm 1, \pm 3, \pm 7, \pm 21$

q's: $\pm 1, \pm 2, \pm 4$

$\frac{p}{q}$'s: $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 3, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm 7, \pm\frac{7}{2}, \pm\frac{7}{4}, \pm 21, \pm\frac{21}{2}, \pm\frac{21}{4}$

$x=5$ is upper bd on real zeros:

5	4	-12	-5	21	-41	-21
		20	40	175	980	4845
	4	8	35	196	976	4824

Bottom row is all nonnegative

$$\begin{array}{r|rrrrr} -1 & 4 & -12 & -5 & 21 & -11 & -21 \\ & & -4 & 16 & -11 & -10 & 21 \\ \hline -1 & 4 & -16 & 11 & 10 & -21 & 0 \\ & & -4 & 20 & -31 & 21 & \\ \hline 3 & 4 & -20 & 31 & -21 & 0 & \\ & & 12 & -24 & 21 & & \\ \hline & 4 & -8 & 7 & 0 & & \end{array}$$

$4x^2-8x+7$

$a=4, b=-8, c=7$

$b^2-4ac=(-8)^2-4(4)(7)$

$=64-112$

$=-48$

$\Rightarrow \sqrt{-48}=\sqrt{-2\cdot2\cdot2\cdot2\cdot3}=2\cdot2i\sqrt{3}=4i\sqrt{3}$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{8\pm4i\sqrt{3}}{2(4)}=\frac{4(2\pm i\sqrt{3})}{8}$

$=\frac{2\pm i\sqrt{3}}{2}=1\pm\frac{i\sqrt{3}}{2}$

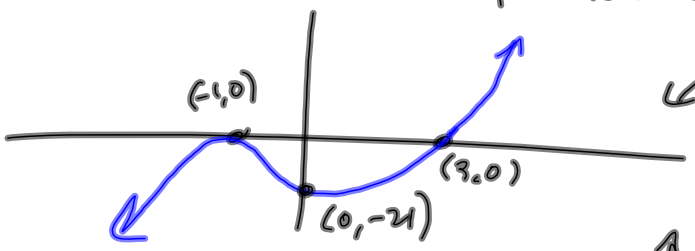
$x=-1\ (m=2),\ 3,\ 1\pm\frac{i\sqrt{3}}{2}$

$2\,|\,48$
$2\,|\,24$
$2\,|\,12$
$2\,|\,6$
3

Factor over the Reals $(x+1)^2(x-3)(4x^2-8x+7)$

Factor over $\mathbb{C}$ $4(x+1)^2(x-3)\left(x-\left(1+\frac{i\sqrt{3}}{2}\right)\right)\left(x-\left(1-\frac{i\sqrt{3}}{2}\right)\right)$

Normal Zeros Have NO expression in the graph



$x=-1$ $m=2$ touches

$x=3$ $m=1$ crosses

y-int: $(0,-21)$

End behavior $4x^5$ ↙ ... ↗

Next up (Min for final)

C5 Solve 2x2 system

$2x+3y=7$

$-5x+4y=8$

Solve for (x,y)

$2x+3y \leq 7$

$-5x+4y \leq 8$

$x \geq 0$

$y \geq 0$

Graph system of linear inequalities

C6 ALL OMITTED

C7 OMIT

C8 $\sum_{k=1}^{n} f(x_k) = f(x_1) + f(x_2) + f(x_3) + \cdots + f(x_n)$

Sigma Notation Basics.

$\sum_{k=1}^{5} (2k-3) = 2(1)-3 + 2(2)-3 + 2(3)-3 + 2(4)-3 + 2(5)-3$

Geometric Series/Sums

$2 + 2\cdot\frac{1}{3} + 2\left(\frac{1}{3}\right)^2 + 2\cdot\left(\frac{1}{3}\right)^3 + \cdots + 2\left(\frac{1}{3}\right)^{49}$

$50 + 50(1.002) + 50(1.002)^2 + 50(1.002)^3 + \cdots$

ANNUITIES $+ 50(1.002)^{49}$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$ Binomial Theorem (Pascal's Triangle)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
0 1 2

Binomial coefficients.

$$(x+y)^5 = \sum_{k=0}^{5} \binom{5}{k} x^{n-k} y^k =$$ coefficients for when you take powers of a binomial $(x+y)^7$

$$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

$\binom{n}{k}$ = n choose k = the # of ways to choose k items from a set of n items.

$\{a,b,c\}$ 3 things, choose 2:

$\{a,b\}$, $\{a,c\}$, $\{b,c\}$ 3 ways $= \binom{3}{2}$
1 2 3

5 choose 2
$\{a,b,c,d,e\}$

$\{a,b\}$, $\{a,c\}$, $\{a,d\}$, $\{a,e\}$, $\{b,c\}$, $\{b,d\}$, $\{b,e\}$,
1 2 3 4 5 6 7
$\{c,d\}$, $\{c,e\}$, $\{d,e\}$
8 9 10

Nov 6-8:26 AM

Our Final is Dec 4th @ 7:10 am

C4: Graphs & Models

Know § P.2, P.3 Exponents I & II

$a \cdot b^{cx+d} + k$ Can you graph this for any a, b, c, d, k?

$a \log_b(cx+d) + k$

The worst we ever got on "a" was $a = -1$

a = coefficient of x inside f(x).

$-3\log_5(-x-5)+7$

STEPS

1 Basic Func.
2 Hor. shift
3 Hor. Flip (if any)
4 Vert. Flip/stretch
5 Vert. shift.

① $\log_5(x)$ (1,0)

② $\log_5(x-5)$ (6,0)

* ③ $\log_5(-x-5)$ (-6,0)

④ $-3\log_5(-x-5)$ (-6,0) $\leftarrow -3 \cdot 0$

⑤ $-3\log_5(-x-5)+7$ (-6,7)

what happens if someone swaps 4 & 5?

④ $\log_5(-x-5)+7$ (-6,7)

⑤ $-3(\log_5(-x-5)+7)$ (-6,-21) Newp.

Nov 6-8:35 AM

Solving & Modeling with Exponentials

Pop was 350 in 1967.
Pop is 600 in 2013

What do you predict the pop is in 2020?
Assume continuous exponential growth.

Let t = # of years after 1967

1967 $\longmapsto$ 1967 − 1967 = 0 $\rightsquigarrow$ (0, 350)

2013 $\longmapsto$ 2013 − 1967 = 46 $\rightsquigarrow$ (46, 600)

2020 $\longmapsto$ 2020 − 1967 = 53 $\rightsquigarrow$ (53, ?)

$$A(t) = Pe^{rt}$$

$$A(0) = 350e^{0 \cdot r} = 350$$

$$\Rightarrow A(t) = 350e^{rt}$$

$$A(46) = 350e^{46r} = 600$$

$$e^{46r} = \frac{60}{35} = \frac{12}{7}$$

$$\ln(e^{46r}) = \ln\left(\frac{12}{7}\right)$$

$$46r = \ln(12) - \ln(7)$$

$$r = \frac{\ln(12) - \ln(7)}{46}$$

Want $A(53) = ?$

$$A(53) = 350e^{53r}$$

$$= 350e^{53 \cdot \frac{\ln(12) - \ln(7)}{46}}$$