

§ 4.3 #s 81, 93, 51, 61

$$\begin{aligned} \textcircled{51} \quad \ln\left(\frac{6\sqrt{x-1}}{5x^3}\right) &= \ln(6) + \ln\sqrt{x-1} - \ln(5) - \ln(x^3) \\ &= \ln(6) + \frac{1}{2}\ln(x-1) - \ln(5) - 3\ln(x) \end{aligned}$$

$$\begin{aligned} &\ln(6\sqrt{x-1}) - \ln(5x^3) \\ &= \ln 6 + \frac{1}{2}\ln(x-1) - [\ln 5 + \ln x^3] \end{aligned}$$

$$\ln 6 + \frac{1}{2}\ln(x-1) - \ln 5 - 3\ln x$$

↓ Take it home ↓

$$\begin{aligned} &\ln(6) + \ln\sqrt{x-1} - \ln 5 - \ln(x^3) \\ &= \ln\left(\frac{6\sqrt{x-1}}{5x^3}\right) \end{aligned}$$

$$\textcircled{61} \quad 3 \log_4(x^2) - 4 \log_4(x^{-3}) + 2 \log_4(x) \quad \text{Cody}$$

$$\begin{aligned} &= \log_4((x^2)^3) - \log_4((x^{-3})^4) + \log_4(x^2) \\ &\quad + \log_4((x^{-3})^{-4}) \end{aligned}$$

$$= \log_4(x^6) - \log_4(x^{-12}) + \log_4(x^2) = \log_4\left(\frac{x^6 \cdot x^2}{x^{-12}}\right)$$

$$= \log_4(x^6 \cdot x^{12} \cdot x^2) = \log_4(x^{20})$$

#81 $(1+r)^3 = 2.3$ Variable in base
is different.

My instinct is to take $\log_{1+r}((1+r)^3)$
but we're not trying to EXTRACT an exponent;
rather, we're trying to get rid of it.

$$\sqrt[3]{(1+r)^3} = \sqrt[3]{2.3}$$

$$1+r = \sqrt[3]{2.3}$$

$$r = \sqrt[3]{2.3} - 1$$

$$\approx .3200$$

	6.614546582
10^Ans	4116675
1257*3275	4116675
2.3^(1/3)-1	.3200061218

98

1960 a gallon of milk cost \$0.49
 2009 \$4.59

Use compound interest to the annual growth rate of the price of a gallon of milk.

Continuous? If not, how many periods/yr?
 4.7% ↓ Annually compounded
m=1

$$A(t) = P\left(1 + \frac{r}{m}\right)^{mt} = P(1+r)^t$$

& we know @ t=0,
we have

Madison

$$A(0) = P(1+r)^0 = P = 0.49$$

$$A(49) = 0.49(1+r)^{49} = 4.59$$

$$(1+r)^{49} = \frac{4.59}{.49}$$

$$\sqrt[49]{(1+r)^{49}} = \sqrt[49]{\frac{4.59}{.49}}$$

$$1+r = \sqrt[49]{\frac{4.59}{.49}}$$

$$r = \sqrt[49]{\frac{4.59}{.49}} - 1$$

$$\approx .0467161145$$

Keli:
Write much
think little.

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(4.59/.49)^(1/49)
)-1
.0467161145
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Cont. Version?

$$A(t) = Pe^{rt}$$

$$A(0) = Pe^{r \cdot 0} = P = .49$$

$$A(t) = .49e^{rt}$$

$$A(49) = .49e^{49r} = 4.59$$

$$e^{49r} = \frac{4.59}{.49}$$

$$\ln(e^{49r}) = \ln\left(\frac{4.59}{.49}\right)$$

$$49r = \ln\left(\frac{4.59}{.49}\right)$$

$$r = \frac{\ln\left(\frac{4.59}{.49}\right)}{49}$$

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ln(4.59/.49)/49
.0456577533
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$$= \frac{\ln(4.59) - \ln(.49)}{49}$$

$$\approx .0456577533$$

$$= 4.56577533\%$$

When in doubt, \downarrow book was ambiguous,
use continuous model.
 Pe^{rt}

S4.4 Equations & Apps.

$$\textcircled{4} \log(x-6) = 1 \qquad \log(16-6) = \log(10) = 1$$

$${}_{10}\log(x-6) = 10^1$$

$$\boxed{x-6=10^1}$$

$$\boxed{x=16}$$

$\textcircled{10}$ weird when base is variable in the log.

$$-\frac{1}{2} = \log_x(9)$$

$$x^{-\frac{1}{2}} = x^{\log_x(9)}$$

$$x^{-\frac{1}{2}} = 9$$

$$\frac{1}{\sqrt{x}} = 9$$

$$1 = 9\sqrt{x}$$

$$\frac{1}{9} = \sqrt{x}$$

$$\left(\frac{1}{9}\right)^2 = (\sqrt{x})^2$$

$$\boxed{\frac{1}{81} = x}$$

$$x^2 = 4$$

$$\sqrt{x^2} = \sqrt{4}$$

$$x = 2$$

$$\sqrt{x^2} = |x|$$

$$(\sqrt{x})^2 = x$$

#4?

$$\textcircled{\#16} \log_6(x-1) + \log_6(x-2) = 1$$

$$\underline{\log_6((x-1)(x-2)) = 1}$$

$${}_6 \log_6((x-1)(x-2)) = {}_6 1$$

$$(x-1)(x-2) = 6$$

$$x^2 - 3x + 2 = 6$$

$$x^2 - 3x - 4 = 0 \quad \neq$$

$$(x-4)(x+1) = 0$$

$$x-4 = 0 \quad \text{OR} \quad x+1 = 0$$

$$\boxed{x=4} \quad \text{OR} \quad \textcircled{x=-1}$$

$$\log_6(x-1) + \log_6(x-2) = 1 \quad \neq \text{ } \mathcal{D}$$

$$\log_6(4-1) + \log_6(4-2) = 1 ?$$

$$\log_6(3) + \log_6(2) = 1 ?$$

$$\log_6(6) = 1 ?$$

$$1 = 1 \quad \checkmark$$

$$\textcircled{\log_6(-1-1) + \dots}$$

→ Not Real

$x = -1 \notin \mathcal{D}$ (question)

$${}_6 \log_6(x-1) + \log_6(x-2) = {}_6 1$$

$${}_6 \log_6(x-1) \quad {}_6 \log_6(x-2) = 6$$

$$(x-1)(x-2) = 6$$

etc.

Radiometric Dating

C-14 has a $\frac{1}{2}$ -life of 5730 yrs.

C-12 is the "natural" state of Carbon.

C-14 has an extra pair of neutrons.

chemically identical to C-12.

But radioactive, from the extra neutrons.

Forms in atmosphere when C-12 is bombarded by Cosmic Rays.

Happens at a steady rate & C-14 in atmosphere is constant.

when you / tree / ... dies, you separate from C-14 cycle. the C-14 is locked in your body & decays.

So something that has 20% as much C-14 in it as found in our atmosphere is how old?

$\frac{1}{2}$ -life is 5730 years.

$$A(t) = Pe^{rt}$$

$$A(5730) = Pe^{5730r} = \frac{1}{2}P \quad \text{Solve for } r$$

$$e^{5730r} = \frac{1}{2}$$

$$\ln(e^{5730r}) = \ln\left(\frac{1}{2}\right) = -\ln 2$$

$$5730r = -\ln 2$$

$$r = -\frac{\ln 2}{5730}$$

So $A(t) = Pe^{-\frac{\ln 2}{5730} t}$

Now to the main question: 20% C-14 is left.

$Pe^{rt} = .2P$

$e^{rt} = .2$

$\ln(e^{rt}) = \ln(.2)$

$rt = \ln(.2)$

$t = \frac{\ln(.2)}{r} = \frac{\ln(.2)}{-\frac{\ln 2}{5730}}$

$= -\frac{5730 \ln(.2)}{\ln(2)} \approx 13,304.64798 \text{ yrs}$

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-5730*ln(.2)/ln(
2)
13304.64798
```

Does this make sense?

t	%	g
0	100	100g
5730	50	50g
11,460	25	25g
17190	12.5	12.5g

20% is here

Misc Probs.

2

$$\begin{aligned}(x-2)(x+1) \\ = x^2 - x - 2\end{aligned}$$

Solve

$$e^{2x} - e^x - 2 = 0$$

is quadratic in form.

Let $u = e^x$

$$e^{2x} = (e^x)^2 \quad \text{Then we have}$$

$$u^2 - u - 2 = 0$$

$$(u-2)(u+1) = 0$$

$$u = -1 \text{ OR } u = 2$$

~~$$e^x = -1$$~~

$$e^x = 2$$

$$\ln(e^x) = \ln(2)$$

$$x = \ln(2)$$

$$e^{2x} - e^x - 2 = 0$$

$$e^{2\ln(2)} - e^{\ln 2} - 2 = 0 ?$$

$$e^{\ln(2^2)} - e^{\ln 2} - 2 = 0 ?$$

$$4 - 2 - 2 = 0 \checkmark$$

§ 4.4 #5 1, 5, 9, 11, 15-21, 26, 27, 30,

41, 43, 49, 53, 59, 63, 65, 68, 75

#21 \emptyset