71

19, 33, 39, 57, 75, 103, 105

$\left(\frac{1}{2}\right)^x = 8 = 2^3 = \left(\left(\frac{1}{2}\right)^{-1}\right)^3 = \left(\frac{1}{2}\right)^{-3}$

$a^x = a^y \Rightarrow x = y$

$x = -3$

71) $8^x = 2$

$\left(2^3\right)^x = 2$

$2^{3x} = 2^1$

$3x = 1$

$x = \frac{1}{3}$

$2 = 8^?$

#103 e. continuously

$Pe^{rt} \approx P\left(1 + \frac{r}{360}\right)^{360t}$

#103

```
5000(1+.08/1)^(1
*6)
      7934.371615
5000(1+.08/4)^(4
*6)
      8042.186247
```

Annually

Quarterly

```
      8042.186247
5000(1+.08/12)^(
12*6)
      8067.510837
5000(1+.08/360)^
(360*6)
      8079.941133
```

Monthly

Daily

```
12*6)
      8067.510837
5000(1+.08/360)^
(360*6)
      8079.941133
5000e^(.08*6)
      8080.372011
```

Continuously.

§4.2 Logarithmic Functions

$\log_a x = y$ means $x = a^y$

SHOW ME YOUR POWER!

$\log_3 81 = 4$ means $81 = 3^4$

Logs & Exponentials are inverses

$\left(\frac{1}{2}\right)^x = 8$ $\qquad$ $8^x = 2$

$\log_{\frac{1}{2}}\left(\left(\frac{1}{2}\right)^x\right) = \log_{\frac{1}{2}}(8)$

$x = \log_{\frac{1}{2}}(8) =$

write argument of log as power of its base

$= \log_{\frac{1}{2}}(2^3)$

$= \log_{\frac{1}{2}}\left(\left(\frac{1}{2}\right)^{-3}\right)$

$= -3$

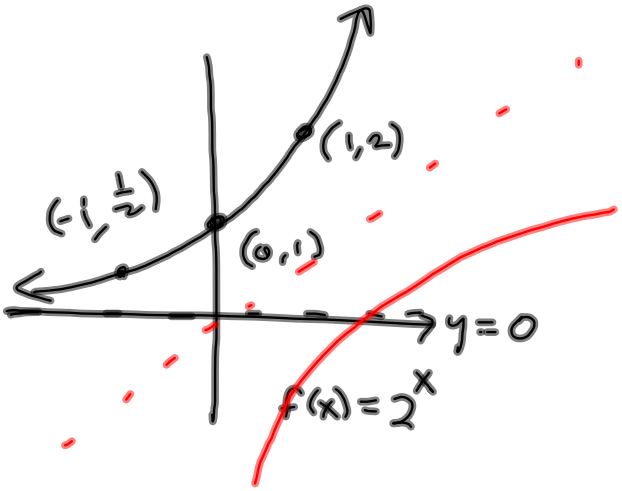
$\log_8(8^x) = \log_8(2)$

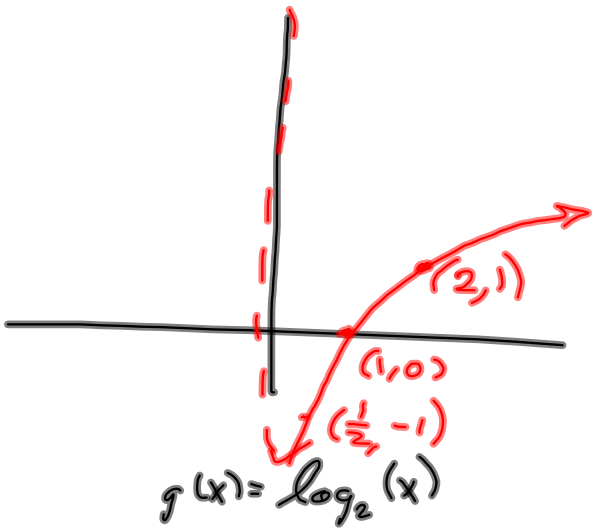
$x = \log_8(2)$

$= \log_8\left(8^{\frac{1}{3}}\right)$

$= \frac{1}{3}$

Calculate w. the Change of Base Formula.





Common log

$\log_{10} x = \log x$

natural log

$\log_e x = \ln x$

$\sqrt[3]{x} = \sqrt{x}$

$\log_a x = M$

$a^{\log_a x} = a^M$

$x = a^M$

$\log_b x = \log_b (a^M)$

$\log_b x = M \log_b (a)$

$\frac{\log_b x}{\log_b a} = M = \log_a x$

To convert $\log_a x$ to $\log_b x$. Change of Base.

$\log_3 (81) = 4$

$\log_3 (3) = 1$

$\log_3 (3^4) = 4$

$4 \log_3 (3) = 4$

Oct 28-8:09 AM

WHY ?

$\log_a = \frac{\log_b x}{\log_b y}$ lets us clobber

things like $\log_{1/2} 8$ with a calculator.

$\log_{1/2} 8 = \frac{\ln 8}{\ln 1/2} = -3$

```
ln(8)/ln(1/2)
                -3
log(8)/log(1/2)
                -3
```

```
ln(1/2)/ln(8)
     -.3333333333
Ans▸Frac
                -1/3
```

$\log_8 (\frac{1}{2})$

$\frac{\ln(1/2)}{\ln(8)} = -\frac{1}{3}$

$\log_8 (2) = \frac{1}{3}$

(9) $2^x = 64 = 2^6$

$x = 6$

$\log_2(2^x) = \log_2(64)$

$x = \log_2(64) = \frac{\ln(64)}{\ln(2)}$

Alternate Approach, anticipating "ln" @ the end.

$2^x = 64$

$\ln(2^x) = \ln(64)$

$x \ln(2) = \ln(64)$

$\frac{x \ln(2)}{\ln(2)} = \frac{\ln(64)}{\ln(2)}$

$x = \frac{\ln(64)}{\ln(2)}$

$\ln(a^x) = x \ln a$

① Write the exponential equation in logarithmic form

② Write the logarithmic equation in exponential form.

$2^x = 64$

$x = \log_2 64$

Solving Equations

Find the APR compounded continuously to nearest 10th of a percent to make $10 grow to $30 in 5 years.

$A = Pe^{rt}$ Continuous Compounding

$30 = 10e^{5r}$

$10e^{5r} = 30$

$e^{5r} = 3$

e^x & $\ln x$ are inverses!

$\ln(e^{5r}) = \ln(3)$

$5r \ln(e) = \ln(3)$

$5r = \ln 3$

$r = \frac{\ln 3}{5} \approx .2197224577$

$= 21.97224577\%$

$= 22.0\%$

$\ln(3)/5$
$.2197224577$

$10^{5r} = 3$

$\ln(10^{5r}) = \ln(3)$

$5r \ln(10) = \ln 3$

$\frac{5r \ln 10}{5 \ln 10} = \frac{\ln 3}{5 \ln 10}$

$r = \frac{\ln 3}{5 \ln 10}$

$10^{5r} = 3$

$\log(10^{5r}) = \log 3$

$5r = \log 3$

$r = \frac{\log 3}{5}$

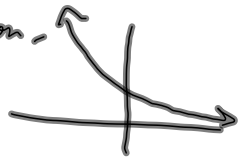
Oct 28-8:39 AM

The $\frac{1}{2}$-life of millsium is 50 yrs.

A sample of millsium has 30% of the original radioactive part left.

How old is the sample?

This is radioactive decay question.



$A = Pe^{rt} = A$

$Pe^{50r} = \frac{1}{2}P$

$\frac{1}{2}$-life is 50 yrs

$\frac{1}{2}$ remains in 50 yrs

$e^{50r} = \frac{1}{2}$

$\ln(e^{50r}) = \ln(\frac{1}{2})$

```
ln(3)/5
     .2197224577
ln(.5)/50
    -.0138629436
```

$50r = \ln(\frac{1}{2})$

$r = \frac{\ln(\frac{1}{2})}{50}$ = relative decay rate.

Kel: $\approx -.0138629436$

Now, how old is the sample?

$Pe^{rt} = .3P$

$e^{rt} = .3$

$\ln(e^{rt}) = \ln(.3)$

$rt = \ln(.3)$

$t = \frac{\ln(.3)}{r} = \frac{\ln(.3)}{\frac{\ln(\frac{1}{2})}{50}}$

≈ 86.84827971

```
ln(3)/5
     .2197224577
ln(.5)/50
    -.0138629436
ln(.3)/Ans
     86.84827971
```

8.4.2 #5 1-8 ALL, 9, 11, 28, 29, 30, 33-43 odds, 47, 53