Keli's question Find all real or imaginary zeros.

55. $f(x) = x^3 - 9x^2 + 26x - 24$ $x = 2, 3, 4$

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12$

$$\begin{array}{r|rrrr} 1 & 1 & -9 & 26 & -24 \\ & & 1 & -8 & 18 \\ \hline & 1 & -8 & 18 & 6 \end{array}$$

$$\begin{array}{r|rr} 4 & 1 & -4 \\ & & 4 \\ \hline & 1 & \end{array}$$

What's this show?

$f(x) = (x-1)(x^2-8x+18) + 6$

$\frac{f(x)}{x-1} = x^2 - 8x + 18 + \frac{6}{x-1}$

$$\begin{array}{r|rrrr} -1 & 1 & -9 & 26 & -24 \\ & & -1 & 10 & -36 \\ \hline & 1 & -10 & 36 & \end{array}$$

This can be thought of as a parabola plus a reciprocal function.

$$\begin{array}{r|rrrr} 2 & 1 & -9 & 26 & -24 \\ & & 2 & -14 & 24 \\ \hline & 1 & -7 & 12 & \end{array}$$

$(x-2)(x^2-7x+12)$

"Away from" $x=1$, $x^2-8x+18$ runs the show.

$$\begin{array}{r|rrr} 2 & 1 & -7 & 12 \\ & & 2 & -10 \\ \hline & 1 & -5 & \end{array}$$

"close to" $x=1$, the $\frac{6}{x-1}$ dominates.

$$\begin{array}{r|rrr} -2 & 1 & -7 & 12 \\ & & -2 & 18 \\ \hline & 1 & -9 & \end{array}$$

$$\frac{x^3-9x^2+26x-24}{x-1}$$

$h(x) = 1x^3 - x^2 - 7x + 15$ $\qquad -3, 2 \pm i$

p: 15

57

q: 1

$\pm 1, \pm 3, \pm 5, \pm 15$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -7 & 15 \\ & & 1 & 0 & -7 \\ \hline & 1 & 0 & -7 & \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -7 & 15 \\ & & -1 & 2 & \\ \hline & 1 & -2 & -5 & \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -7 & 15 \\ & & 3 & 6 & \\ \hline & 1 & 2 & -1 & \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & -1 & -7 & 15 \\ & & -3 & 12 & -15 \\ \hline & 1 & -4 & 5 & 0 \\ & x^2 & x & c & r \end{array}$$

$\boxed{x = -3}$

$\boxed{(x+3)(x^2-4x+5)}$

Because x^2-4x+5 has no real roots, this is as far as f(x) factors over the real number field.

We say x^2-4x+5 is an irreducible quadratic factor.

Depressed Equation

$x^2 - 4x + 5 = 0$

$x^2 - 4x + 2^2 = -5 + 4$

$(x-2)^2 = -1$ Paul Da Man

$x - 2 = \pm\sqrt{-1} = \pm i$

$\boxed{x = 2 \pm i}$

$(x+3)(x-(2+i))(x-(2-i))$ is f(x) split into linear factors. Factored over the complex Numbers.

LAST TIME

Bounds on Real Zeros

Descartes' Rule of Signs

These two help us eliminate possibilities when we're breaking things down.

Bounds:

$f(x) = x^3 - x^2 - 7x + 15$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -7 & 15 \\ & & 1 & 0 & -7 \\ \hline & 1 & 0 & -7 & 8 \end{array}$$

Nada

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -7 & 15 \\ & & 3 & 6 & -3 \\ \hline & 1 & 2 & -1 & \end{array}$$

$$\begin{array}{r|rrrr} 5 & 1 & -1 & -7 & 15 \\ & & 5 & 20 & 65 \\ \hline & 1 & 4 & 13 & 80 \end{array}$$

Because this row is all positives, we don't have to check $x = 15$.

5 is an upper bound on real zeros.

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$$\begin{array}{r|rrrr} -3 & 1 & -1 & -7 & 15 \\ & & -3 & 12 & -15 \\ \hline & 1 & -4 & 5 & 0 \\ & x^2 & x & c & r \end{array}$$

Bottom Row alternates, so $x=-3$ is a LOWER bound on real zeros.

No separate test question on this.
But it can save time on a bigger problem.

Descartes' Rule of Signs

$f(x) = 8x^3 - 36x^2 + 46x - 15$ 3 or 1 positive zeros

$f(-x) = -8x^3 - 36x^2 - 46x - 15$ 0 negative zeros.

$f(x) = x^4 + 2x^3 - x^2 - 4x - 2$ 1 pos. zero

$f(-x) = x^4 - 2x^3 - x^2 + 4x - 2$ 3 or 1 neg. zero.

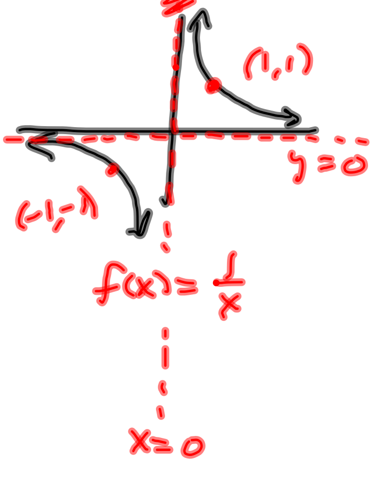
79 $f(x) = \frac{2x+1}{x-2}$ See instructions

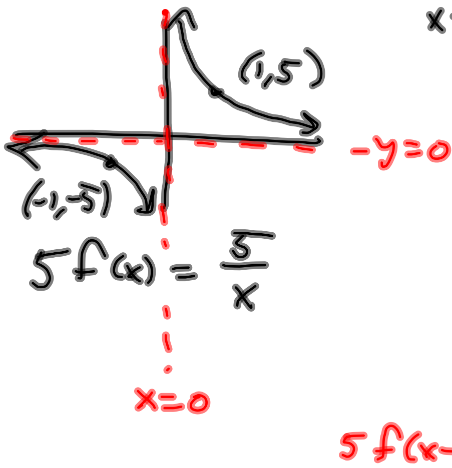
$$\begin{array}{r|rr} 2 & 2 & 1 \\ & & 4 \\ \hline & 2 & 5r \end{array} \qquad f(x) = 2 + \frac{5}{x-2}$$

Coming Soon: Graphs of Rational Functions.

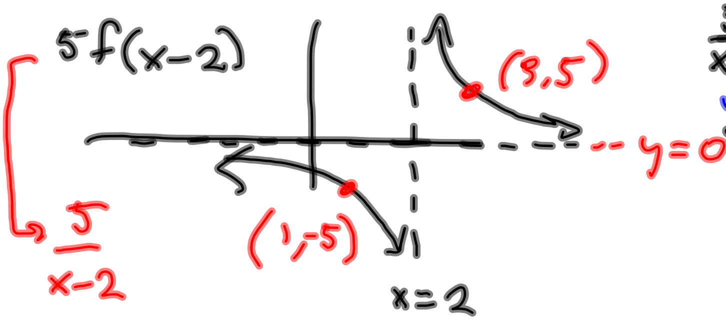
This trick allows us to "see" this one as a transformation on $f(x) = \frac{1}{x}$

$$\frac{2x+1}{x-2} = 2 + \frac{5}{x-2} \qquad \frac{1}{x} \longrightarrow \frac{5}{x} \longrightarrow \frac{5}{x-2} \longrightarrow \frac{5}{x-2} + 2$$

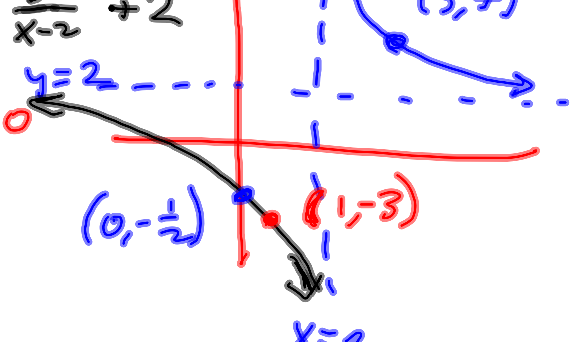




$f(0) = -\frac{1}{2}$



$5f(x-2)+2$



§ 3.5 #s 53-56 ALL, 73, 74, 85-95 odds

Skip 3.4 for a time.

Book says "Test Point" we use one test point or end behavior and analyze sign changes at x-ints (3.5) and vertical Asymptotes (3.6)

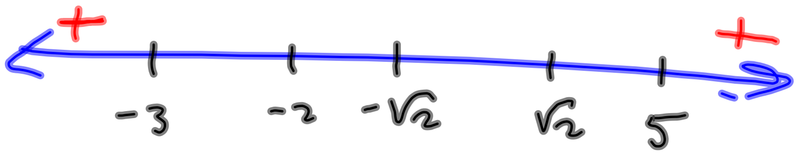
Rough sketches of factored polynomials

$$(x+3)(x-5)^2(x+2)^3(x-\sqrt{2})(x+\sqrt{2})$$

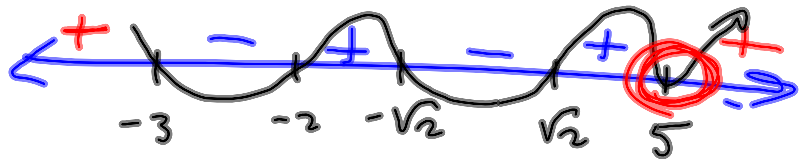
Quick sketch.

End Behavior (large x):

$$+(x)(x)^2(x)^3(x)(x) = +x^8$$



Just from end behavior



Solve $(x+3)(x-5)^2(x+2)^3(x-\sqrt{2})(x+\sqrt{2}) \geq 0$

$$(-\infty,-3] \cup [-2,-\sqrt{2}] \cup [\sqrt{2},\infty)$$

$$(x+3)(x-5)^2(x+2)^3(x-\sqrt{2})(x+\sqrt{2}) > 0$$

$$(-\infty,-3) \cup (-2,-\sqrt{2}) \cup (\sqrt{2},5) \cup (5,\infty)$$

Not > 0 @ $x=5$

$$(x+3)(x-5)^2(x+2)^3(x-\sqrt{2})(x+\sqrt{2}) < 0$$

$$(-3,-2) \cup (-\sqrt{2},\sqrt{2})$$

$$(x+3)(x-5)^2(x+2)^3(x-\sqrt{2})(x+\sqrt{2}) \leq 0$$

$$[-3,-2] \cup [-\sqrt{2},\sqrt{2}] \cup \{5\}$$

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I §3.6 #s 33-56, 61-71 ODDS

II §3.6 #s 95-113 ODDS.

Homework.

Oct 9-8:58 AM