§3.2

Remainder Thm Questions #s 23-34

23 $f(x) = x^5 - 1$

$f(1)$:

1	1	0	0	0	0	-1
		1	1	1	1	1
	1	1	1	1	1	0
	4	3	2	1	c	r

Moving on in lecture...

This work shows

$$\boxed{x^5 - 1 = (x-1)(x^4 + x^3 + x^2 + x + 1)}$$

$$\boxed{f(1) = 0}$$

§3.2 #77 Book wants to see the zeros.
I want to see zeros & the factorization.

$$f(x) = \underbrace{(x^2-4x+1)}_{2} + \underbrace{(x^3-9x^2+23x-15)}_{3} = 5$$

Split $f(x)$ into Linear Factors.

$x^2-4x = -1$

$x^2-4x+2^2 = -1+4$

$(x-2)^2 = 3$

$x-2 = \pm\sqrt{3}$

$\boxed{x = 2\pm\sqrt{3}}$

$x^3-9x^2+23x-15 = 0$

Rat'l Zeros:

p: -15

q: 1

$\frac{p}{q}$: $\pm1, \pm3, \pm5, \pm15$

1. Find a zero "c"
2. Factor out "x−c"
3. Repeat, on "depressed" equation.

$x=1$

$$\begin{array}{r|rrrr} 1 & 1 & -9 & 23 & -15 \\ & & 1 & -8 & 15 \\ \hline & 1 & -8 & 15 & \boxed{0} \end{array}$$

Sweet!

This gives $(x-1)(x^2-8x+15)$

This takes us down to a quadratic polynomial $x^2 - 8x + 15$

$$x^2 - 8x \quad = -15$$
$$x^2 - 8x + 4^2 = -15 + 16$$
$$(x-4)^2 = 1$$

$$x - 4 = \pm\sqrt{1} = \pm 1$$

$$x = 4 \pm 1 \to 5,\ 3$$

$$\boxed{x = 1, 3, 5}$$

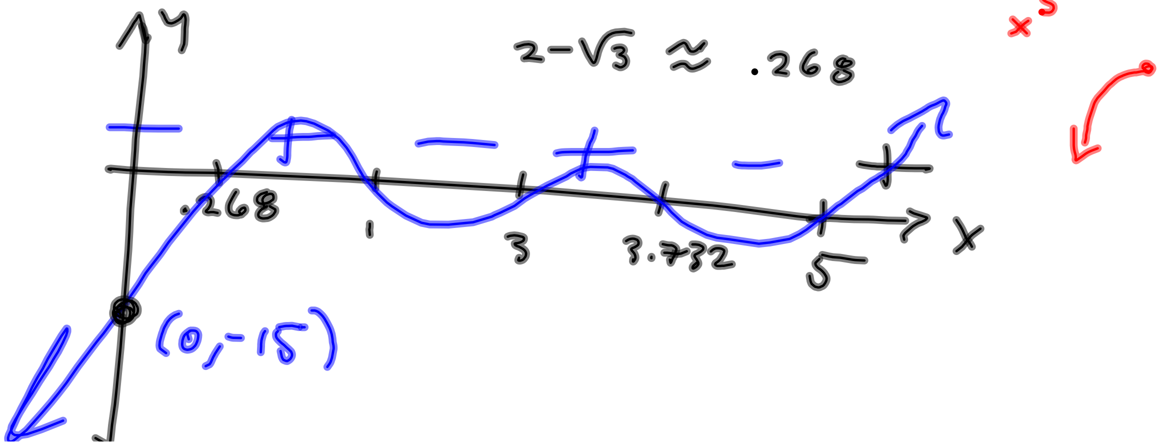
$$\boxed{x = 1, 3, 5, 2 \pm \sqrt{3}}$$ 5 zeros.

On the test, I'll ask to see "factored form": $(x-1)(x-3)(x-5)(x-(2+\sqrt{3}))(x-(2-\sqrt{3}))$

Further Analysis: The PICTURE for this is:

$2+\sqrt{3} \approx 3.732$

$2-\sqrt{3} \approx .268$



57 $x^3-x^2-7x+15$ #s 55-78 Find all real & imaginary zeros.

Rat:

$\frac{p}{q}$: $\pm 1, \pm 3, \pm 5, \pm 15$

$\sqrt{3}\,i$ is imaginary

$\boxed{2+\sqrt{3}\,i \text{ is nonreal complex.}}$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -7 & 15 \\ & & 1 & 0 & -7 \\ \hline & 1 & 0 & -7 & 8 \end{array}$$ Newp.

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -7 & 15 \\ & & -1 & 2 & 5 \\ \hline & 1 & -2 & -5 & 20 \end{array}$$ Nope.

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -7 & 15 \\ & & 3 & 6 & -3 \\ \hline & 1 & 2 & -1 & \end{array}$$ Meh

$$\begin{array}{r|rrrr} -3 & 1 & -1 & -7 & 15 \\ & & -3 & 12 & -15 \\ \hline -3 & 1 & -4 & 5 & 0 \end{array}$$ Sweet!

$x=-3$

$(x+3)(x^2-4x+5)$

$x^2-4x+5=0$

$b^2-4ac=(-4)^2-4(1)(5)$

$=16-20$

$=-4$

$x=\frac{4\pm\sqrt{-4}}{2(1)}=\frac{4\pm 2i}{2}=2\pm i$

$\boxed{\text{Zeros: } -3,\ 2\pm i}$ All book asked.

$(x+3)(x-(2+i))(x-(2-i))=f(x)$

$2+i$ is <u>NOT</u> imaginary.

It's NONREAL, with an imaginary <u>part</u> and a real part.

#67

$x^4+2x^3-x^2-4x-2$

$\frac{p}{q}: \pm 1, \pm 2$

$$\begin{array}{r|rrrrr} 1 & 1 & 2 & -1 & -4 & -2 \\ & & 1 & 3 & 2 & -2 \\ \hline & 1 & 3 & 2 & -2 & -4 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 1 & 2 & -1 & -4 & -2 \\ & & -1 & -1 & 2 & 2 \\ \hline & 1 & 1 & -2 & -2 & 0 \end{array}$$ sweet!

check $x=-1$, again

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -2 & -2 \\ & & -1 & 0 & 2 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$ sweet!

$\sqrt{2}^2=2$

$x^2-2=(x-\sqrt{2})(x+\sqrt{2})$

$(x+1)^2(x^2-2)$ is where we are.

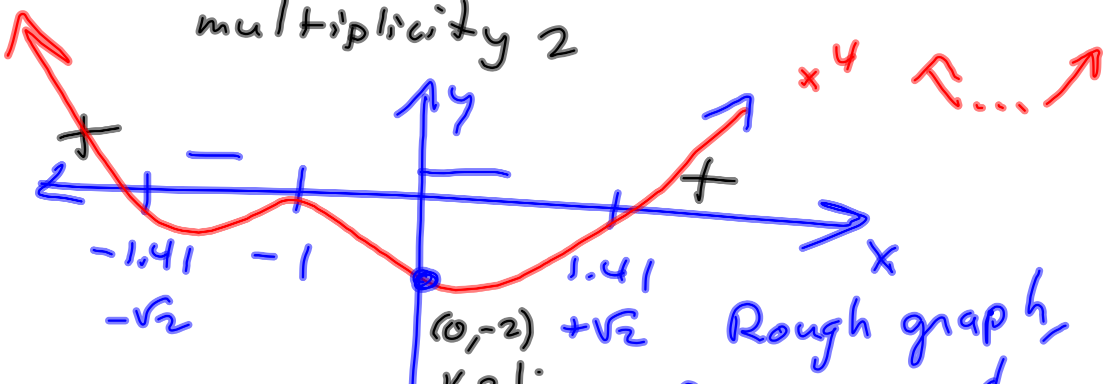
$x^2-2=0$

$x^2=2$

$\boxed{x=\pm\sqrt{2}, -1 \text{ BOOK ANSWER}}$

$(x+1)^2(x-\sqrt{2})(x-(-\sqrt{2}))$

$x=-1$ is a zero of multiplicity 2



Keli:
Dq
Woman

Rough graph from zeros & End Behavior.

Descartes Says...

$\ldots + \underbrace{x^3 -}_{1} 5x^2 - 4x - 2 = f(x)$ — $\boxed{\text{has exactly one positive root.}}$

$\ldots -x^3 - 5x^2 + 4x - 2 = f(-x)$ — two or zero negative

$x^5 + x^4 - 5x^3 + 7x - 3$ — 3 or 1 positive roots

$-x^5 + x^4 + 5x^3 - 7x - 3 = f(-x)$ — 2 or 0 negative

$x^5 + x^4 + 37x^3 + 5x^2 + x + 11$

Oct 7-8:53 AM

§3.3 #5 15, 17, 19, 23, 31, 33, 39

$\boxed{\text{§3.2 Due on Wed.}}$

Oct 7-9:00 AM