§3.1 #s 65–76–type problems.

(65) $x^2 - 4x + 2 < 0$ ("−")

$x^2 - 4x + 2 = 0$

$a = 1, b = -4, c = 2$

$b^2 - 4ac = (-4)^2 - 4(1)(2)$

$= 16 - 8$

$= 8$

$\sqrt{4 \cdot 2} = \sqrt{4}\sqrt{2}$

$= 2\sqrt{2}$

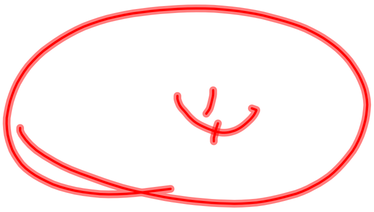
$\sqrt{8} = \sqrt{2 \cdot 2 \cdot 2} = 2\sqrt{2}$

So $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-4) \pm 2\sqrt{2}}{2(1)} = \dfrac{4 \pm 2\sqrt{2}}{2} = \dfrac{2(2 \pm \sqrt{2})}{2} = 2 \pm \sqrt{2}$

$+ \quad - \quad +$

N $2-\sqrt{2}$ Y $2+\sqrt{2}$



Test Values Method

Our work showed we have this factorization: $(x - (2+\sqrt{2}))^1 (x - (2-\sqrt{2}))^1$

$x = 4$: $f(x) = 4^2 - 4(4) + 2 = +2$

Sign changes when we cross $x = 2+\sqrt{2}$, because it's $(x-(2+\sqrt{2}))^1$, and 1 is odd.

Final Ans: $x \in (2-\sqrt{2}, 2+\sqrt{2})$

$ax^2+bx+c=0 \Longrightarrow$

b^2-4ac = Discriminant

and $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Vertex = (h,k), where

$h=-\dfrac{b}{2a}$, $k=f\left(-\dfrac{b}{2a}\right)$

Completing the square:

$ax^2+bx+c = a(x-h)^2+k$

ANY TIME YOU FIND THE zeros of a quadratic, you're implicitly finding how it factors:

$$a\left(x-\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(x-\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$(3x+2)(2x-3) = 6x^2-5x-6$

Oct 4-8:00 AM

Solve $6x^2-5x-6 \geq 0$

$a=6,\ b=-5,\ c=-6$

$$b^2-4ac = (-5)^2-4(6)(-6)$$
$$= 25 + 144$$
$$= 169$$

$\sqrt{169} = 13$

$$x = \frac{5 \pm 13}{2(6)} \quad \begin{cases} \frac{18}{12} = \frac{3}{2} \\ -\frac{8}{12} = -\frac{2}{3} \end{cases}$$

check:

This gives us

$$6\left(x-\tfrac{3}{2}\right)\left(x-\left(-\tfrac{2}{3}\right)\right) = f(x)$$

$+ \quad - \quad +$

$Y \quad -\frac{2}{3} \quad N \quad \frac{3}{2} \quad Y \qquad \geq 0$

$$x \in \boxed{\left(-\infty, -\tfrac{2}{3}\right] \cup \left[\tfrac{3}{2}, \infty\right)}$$

Fundamental Theorem of Algebra says we can split any polynomial into linear factors.

we've been doing this like crazy for quadratics.

$f(x) = x^2 - 4x + 1$

$a = 1,\ b = -4,\ c = 1$

$b^2 - 4ac = (-4)^2 - 4(1)(1)$

$= 16 - 4$

$= 12$

$\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

So $f(x) = 1(x - (2+\sqrt{3}))(x - (2-\sqrt{3}))$

This is f(x) split into Linear factors.

Oct 4-8:15 AM

Rational Zeros Theorem

If $x = \frac{p}{q}$ is a zero of a polynomial, then q is a factor of the leading term, and p is a factor of the constant term

$\cancel{(3x+7)(2x-5)} = 6x^2 - x - 35$

zeros : $\cancel{-\frac{7}{3},\ \frac{5}{2}}$

$2x-5=0$, etc.

$\frac{5}{2}$ is a zero,

5 is a factor of -35,

2 is a factor of 6.

Oct 4-8:21 AM

Test question: Split

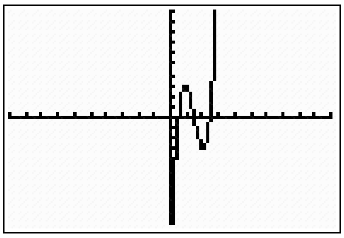
$f(x) = 8x^3 - 36x^2 + 46x - 15$ into linear factors.

$\frac{p}{q} = \pm 1, \pm 15, \pm \frac{15}{2}, \pm \frac{15}{4}, \pm \frac{15}{8}$

$\pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm \frac{3}{8},$

$\pm 5, \pm \frac{5}{2}, \pm \frac{5}{4}, \pm \frac{5}{8}$

ON A TEST, No graphing calculator. On homework, use 'em like crazy.



I'll guess $x = \frac{1}{2}$

$$\begin{array}{c|cccc} \frac{1}{2} & 8 & -36 & 46 & -15 \\ & & 4 & -16 & 15 \\ \hline & 8 & -32 & 30 & 0 \end{array}$$

$f(x) = (x - \frac{1}{2})(8x^2 - 32x + 30)$

To finish, all we need to do is factor the depressed polynomial

$8x^2 - 32x + 30$

$= 2(4x^2 - 16x + 15)$ $a = 4,\ b = -16,\ c = 15$

$b^2 - 4ac = (-16)^2 - 4(4)(15)$

Phillip Da Man

$= 256 - 240$

$= 16$

$x = \dfrac{16 \pm \sqrt{16}}{2(4)} = \dfrac{16 \pm 4}{8} = \dfrac{4 \pm 1}{2}$ $\to \dfrac{5}{2}$, $\to \dfrac{3}{2}$

~~So $f(x) = 8(x - \frac{1}{2})(x - \frac{5}{4})(x - \frac{3}{4})$~~

$f(x) = 8(x - \frac{1}{2})(x - \frac{5}{2})(x - \frac{3}{2})$

$= 2(x - \frac{1}{2})(2)(x - \frac{5}{2})(2)(x - \frac{3}{2})$

$= (2x-1)(2x-5)(2x-3)$

$= (2x-1)(4x^2 - 16x + 15)$

$$
\begin{array}{rrrr}
= 8x^3 & -32x^2 & +30x & \\
 & -4x^2 & +16x & -15 \\
\hline
8x^3 & -36x^2 & +46x & -15
\end{array}
$$

Bounds on Real Zeros

Descartes Rule of Signs

These help us eliminate possibilities without having to guess and check every possibility.

Bounds on Real Zeros

Check $x=3$ $\boxed{x-3}$

$$\begin{array}{r|rrrr} 3 & 8 & -36 & 46 & -15 \\ & & 24 & -36 & 30 \\ \hline & 8 & -12 & 10 & 15 \end{array}$$

Inconclusive,

$$\begin{array}{r|rrrr} 4 & 8 & -36 & 46 & -15 \\ & & 32 & & \\ \hline & 8 & -4 & & \end{array}$$

Nope.

$$\begin{array}{r|rrrr} 5 & 8 & -36 & 46 & -15 \\ & & 40 & 20 & \text{Big} + \\ \hline & 8 & 4 & 66 & \text{Big} + \end{array}$$

This says look no further than $x=5$ for positive roots

$$\frac{p}{q} = \pm 1,\ \cancel{\pm 15},\ \cancel{\pm \frac{15}{2}},\ \pm \frac{15}{4},\ \pm \frac{15}{8}$$

$$\pm 3,\ \pm \frac{3}{2},\ \pm \frac{3}{4},\ \pm \frac{3}{8},$$

$$\cancel{\pm 5},\ \pm \frac{5}{2},\ \pm \frac{5}{4},\ \pm \frac{5}{8}$$

Lower Bounds on Negative Zeros:

$$
\begin{array}{r|rrrr}
-5 & 8 & -36 & 46 & -15 \\
 & & -40 & +\text{BIG} & -\text{Huge} \\
\hline
 & 8 & -76 & +\text{BIG} & -\text{Huge}
\end{array}
$$

Signs alternate. This means -5 is a lower bound on real zeros.

$$\frac{p}{q} = \pm 1, \pm 15, \pm \frac{15}{2}, \pm \frac{15}{4}, \pm \frac{15}{8}$$

$$\pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm \frac{3}{8},$$

$$\pm 5, \pm \frac{5}{2}, \pm \frac{5}{4}, \pm \frac{5}{8}$$