#43 $f(x) = x^2 - x$

vertex, aos, ... Graph

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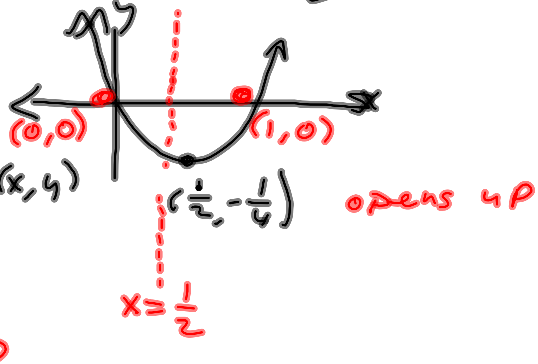
$\boxed{M1}$ $f(x) = x^2 - x$

$= x^2 - x + \left(\frac{1}{2}\right)^2 - \frac{1}{4}$

$= \left(x - \frac{1}{2}\right)^2 - \frac{1}{4}$

$(h,k) = \left(\frac{1}{2}, -\frac{1}{4}\right)$

AOS : $x = \frac{1}{2}$




SET = 0 $\Rightarrow$

$\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} = 0$

Need 2 see — $\left(x - \frac{1}{2}\right)^2 = \frac{1}{4}$

$\left.\begin{aligned} \sqrt{\left(x - \tfrac{1}{2}\right)^2} &= \sqrt{\tfrac{1}{4}} \\ \left|x - \tfrac{1}{2}\right| &= \tfrac{1}{2} \end{aligned}\right\}$ Typically skipped.

Need 2 see — $x - \frac{1}{2} = \pm \frac{1}{2}$

$x = \frac{1}{2} \pm \frac{1}{2}$ → $1 \rightarrow (1,0)$

→ $0 \rightarrow (0,0)$

$\boxed{M2}$ $x^2 - x = 0$ If you can't factor it,

$x(x-1) = 0$ $b^2 - 4ac = (-1)^2 - 4(1)(0) = 1$

$x = 0, 1$ $x = \frac{-(-1) \pm \sqrt{1}}{2(1)} = \frac{1 \pm 1}{2} \to 1, 0$ etc.

M2a For parabola, vertex is $\frac{1}{2}$-way between x-ints.

$\frac{0+1}{2} = \frac{1}{2} = h$ $(h,k) = (\frac{1}{2}, -\frac{1}{4})$

$f(\frac{1}{2}) = (\frac{1}{2})^2 - \frac{1}{2} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} = k$

M2b $a = 1, b = -1, c = 0$

$-\frac{b}{2a} = \frac{1}{2} = h$ $(h,k) = (\frac{1}{2}, -\frac{1}{4})$

$f(-\frac{b}{2a}) = (\frac{1}{2})^2 - \frac{1}{2} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} = k$

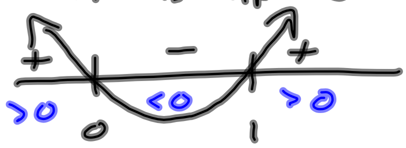
Recall Quadratic Inequalities.

(1) Graphical Method

$x^2 - x < 0$

We know the zeros are $x = 0, 1$

opens up




$< 0$

want "—"

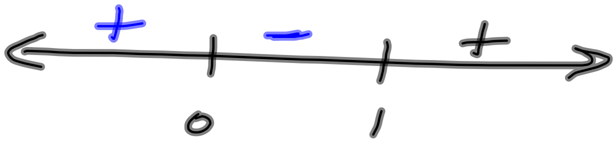
$\Longrightarrow x \in (0,1) = \{x \mid 0 < x < 1\}$

(2) New! Test value method

(3) End behavior, intercepts, and multiplicity

(2) ... $x = 0, 1$ are the zeros of $x^2 - x$

So, to solve $x^2 - x > 0$, lay out the zeros on a number line.




$(-\infty, 0)$ $-1$ $(-1)^2 - (-1) = 1 + 1 = 2$ $+$

$(0, 1)$ $\frac{1}{2}$ $\left(\frac{1}{2}\right)^2 - \frac{1}{2} = -\frac{1}{4}$ $-$

$(1, \infty)$ $2$ $2^2 - 2 = 4 - 2 = 2$ $+$

FROM THE SIGN PATTERN

$x^2 - x > 0$

$x(x-1) > 0$

$x > 0$ OR $x - 1 > 0$

is what idiots do

Nooooooooo.... !!!

!!!!!!

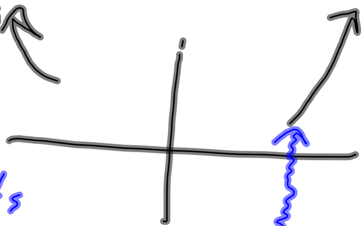
Method (3)

$x^2 - x \geq 0$

$x(x-1) \geq 0$

$x^2 - x$ is a parabola opens up.

End behavior:



So sign pattern starts from this

$+$

0 1

what happens @ $x=1$?

Touch or cross? Crosses

$-$ $+$

1

@ $x=0$, it crosses again.

$+$ $-$ $+$

$$x^1(x-1)$$

1 is odd. crosses the x-axis @ $x=0$

$$x(x-1)^1$$

1 is odd. crosses the x-axis @ $x=1$.

$x=0$ is a zero of multiplicity 1. So is $x=1$.

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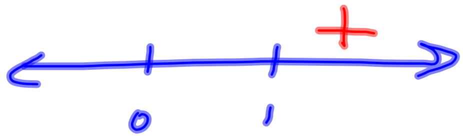
Solve

$$x^2(x-1)^5 \ge 0$$

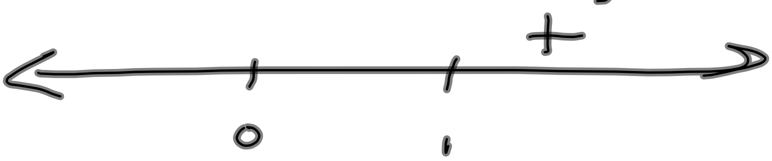
$$(\text{Big})^2(\text{Big}-1)^5 = +$$

$$100^2(99)^5$$



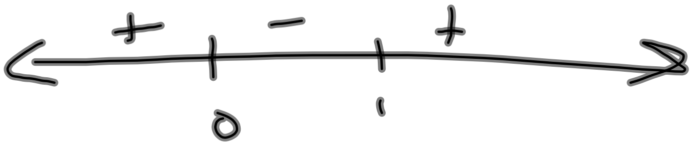

End Behavior: $x = \text{Big} + \Longrightarrow$

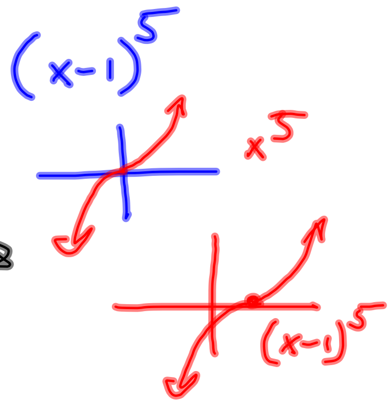
$$(+^2)(+^5)$$




what happens @ $x=1$?

5 is odd Cross.



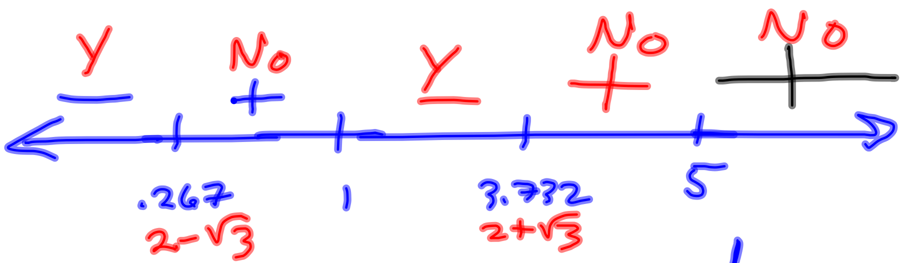

@ $x=0$?

2 is even doesn't cross.

because $x^2$ doesn't cross the x-axis.

$2+\sqrt{3} \approx 3.732$

$2-\sqrt{3} \approx .267$




$$(x-5)^2(x-1)^3(x-(2+\sqrt{3}))(x-(2-\sqrt{3}))^1 \le 0$$

$$(\text{Big})^2(\text{Big})^3(\text{Big})(\text{Big})$$

$(-\infty, 2-\sqrt{3}] \cup [1, 2+\sqrt{3}]$

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# §3.2 Zeros of Polynomials.

Synthetic Division

$$\frac{x^2+5x-7}{x-2} \quad : \quad \begin{array}{r|rrr} 2 & 1 & 5 & -7 \\ & & 2 & 14 \\ \hline & 1 & 7 & 7 \end{array}$$

7 = Remainder.

Let $f(x) = x^2+5x-7$.

Then $f(2)=7$, by Remainder Theorem

The work, above can be interpreted a couple ways:

① $\dfrac{x^2+5x-7}{x-2} = x+7+\dfrac{7}{x-2}$ See exercises @ end of §3.2.

③ $x^2+5x-7 = \underbrace{(x-2)(x+7)+7}$

$f(x) = x^2+5x-7$

$\Longrightarrow$

$f(2)=7$

So you SEE why the remainder theorem works. @ $x=2$, only the remainder remains.

$f(x) = 5x^4 - 3x^3 + 17x - 11$

Find $f(3)$:

$$
\begin{array}{r|rrrrr}
3 & 5 & -3 & 0 & 17 & -11 \\
  &   & 15 & 36 & 108 & 375 \\
\hline
  & 5 & 12 & 36 & 125 & \boxed{364 = f(3)}
\end{array}
$$

$5(3)^4 - 3(3)^3 + 17(3) - 11$

$= 5(81) - 81 + 51 - 11$

$4(81) + 40$

$324 + 40 = 364$ Checks!

That was Remainder Theorem.

FACTOR THEOREM

is the Remainder Theorem in the happy situation where Remainder = 0.

$$(x-(2+\sqrt{3}))(x-(2-\sqrt{3}))$$

$$= x^2-(2-\sqrt{3})x-(2+\sqrt{3})x+\overset{(a-b)(a+b)}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$= x^2-2x+\sqrt{3}x-2x-\sqrt{3}x+\overset{a^2-b^2}{4-3}$$

$$= x^2-4x+1$$

Find zeros of $f(x)=x^2-4x+1$ and factor it.

We built f(x) from a desired factorization.
Let's reclaim the factorization.

$x^2-4x+1=0$

$a=1, b=-4, c=1$

$b^2-4ac=(-4)^2-4(1)(1)$
$=16-4$
$=12$

$$x=\frac{4\pm 2\sqrt{3}}{2(1)} = \frac{4\pm 2\sqrt{3}}{2} = \frac{2(2\pm\sqrt{3})}{2} = 2\pm\sqrt{3}$$

So, $\sqrt{12}=\sqrt{4\cdot 3}=\sqrt{4}\sqrt{3}=2\sqrt{3}$

$2\pm\sqrt{3}$ are the zeros. FACTOR THEOREM says $(x-(2+\sqrt{3}))(x-(2-\sqrt{3}))=f(x)$.

§ 3.2 #s $\boxed{9-15, 23-29, 35-41}$ (ODDS), 47, 49, 55, 57, 58, 61, 67, 77, 79, 81, 83

§ 3.2 hand-in Friday @ end of class.

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