Loose Leaf Notebook

Outline C3

Pg 258 The graph of any quadratic function is a transformation on the graph of $f(x)=x^2$

completing the square is the path.

$$f(x) = x^2+4x-7$$

$$= x^2+4x+2^2-4-7$$

$$= (x+2)^2-11$$

I'm not solving an equation.
I'M MANIPULATING AN EXPRESSION.

vertex: $(-2,-11)$

Zeros: $(x+2)^2-11 = 0$

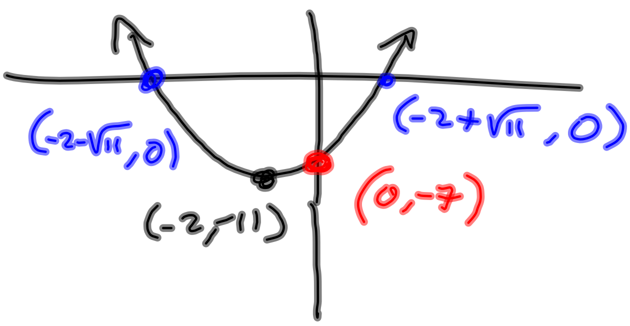
$$(x+2)^2 = 11$$

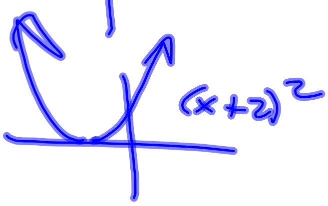
$$x+2 = \pm\sqrt{11}$$

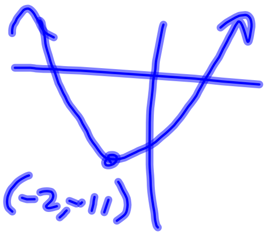
$$x = -2\pm\sqrt{11}$$

$(h,k) = (-2,-11)$

A COMPLETE GRAPH:







In general,

$f(x) = ax^2 + bx + c$

$= a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$

$\Rightarrow \frac{1}{a}f(x) = x^2 + \frac{b}{a}x + \frac{c}{a}$

$= x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}$

$= \frac{1}{a}f(x) = \left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2}$

$\Rightarrow f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$

See Proof, pg 258. Book does it one way. I'll do it slightly differently.

$\frac{\frac{b}{a}}{2} = \frac{1}{2} \cdot \frac{b}{a} = \frac{b}{2a}$

$\left(\frac{b}{2a}\right)^2 = \frac{b^2}{2^2a^2}$

Scratch:

$-\frac{b^2}{4a^2} + \frac{c}{a} \cdot \frac{4a}{4a}$

$= \frac{-b^2 + 4ac}{4a^2}$

$= \frac{4ac - b^2}{4a^2}$

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Pg 260 The vertex of $ax^2+bx+c = f(x)$ is $(h,k) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$$ax^2+bx+c = a(x-h)^2+k$$

$$= a\left(x+\frac{b}{2a}\right)^2 + f\left(-\frac{b}{2a}\right)$$

The "cheat" for completing the square.

$$\Rightarrow f(x) = a\left(x+\frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a}$$

→ According to Steve, this MUST be $f\left(-\frac{b}{2a}\right)$

$$f\left(-\frac{b}{2a}\right) = a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c$$

$$= a\left(\frac{b^2}{4a^2}\right) - \frac{b^2}{2a} + c$$

$$= \frac{b^2}{4a} - \frac{b^2}{2a} + \frac{c}{1}$$

$$= \frac{b^2}{4a} - \frac{2b^2}{4a} + \frac{4ac}{4a} = -\frac{b^2}{4a} + \frac{4ac}{4a}$$

$$= \frac{4ac-b^2}{4a}$$

METHOD 1: $f(x) = x^2 + 4x - 7$

Did this. $= x^2 + 4x + 2^2 - 4 - 7$

$= (x+2)^2 - 11$

METHOD 2: $f(x) = x^2 + 4x - 7$

$a = 1, b = 4, c = -7$

$-\frac{b}{2a} = -\frac{4}{2} = -2 = h$

$f(-\frac{b}{2a}) = (-2)^2 + 4(-2) - 7$

$= 4 - 8 - 7$

$= -11 = K$

So, $f(x) = (x-(-2))^2 - 11$

$= (x+2)^2 - 11$

$\frac{4ac - b^2}{4a} = K = \frac{4(1)(-7) - 4^2}{4(1)} = \frac{-28-16}{4} = -\frac{44}{4} = -11$

You COULD memorize this, but I never did.

Pg 21

$f(x) = ax^2+bx+c = 0 \implies x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x^2+4x-7 = 0$

$a=1, b=4, c=-7$

$b^2-4ac = 4^2-4(1)(-7) = 16+28 = 44$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{44}}{2(1)}$

$2 \,|\, 44$

$2 \,|\, 22$

11

pair of deuces

$= \frac{-4 \pm 2\sqrt{11}}{2}$

$= \frac{2(-2 \pm 1\sqrt{11})}{2}$

$= -2 \pm \sqrt{11}$

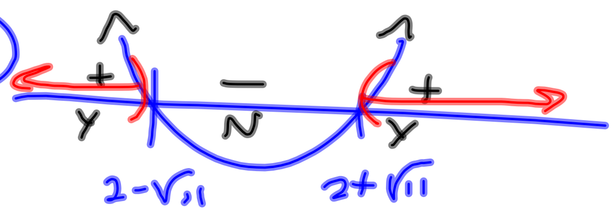
Pg 262 $x^2+4x-7>0$

Solving Quadratic Inequalities graphically.

$ax^2+bx+c>0$ (or $\geq$, $<$, $\leq$)

1. 0 on one side. $x^2+4x-7=0$
2. Solve $f(x)=0$ $x=2\pm\sqrt{11}$
3. Graph it (Rough sketch w/ intercepts)
4. Read Solution

③

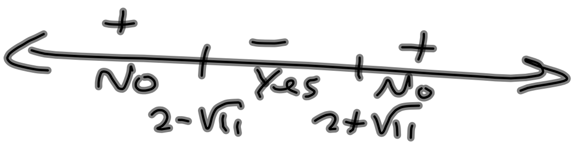


$x^2+4x-7>0$

">0" Positive +

$(-\infty, 2-\sqrt{11}) \cup (2+\sqrt{11}, \infty)$

Another one; $x^2+4x-7\leq 0$



Same sign pattern.

$[2-\sqrt{11}, 2+\sqrt{11}]$

"≤ 0" Negative −

Pg 263 Solving $ax^2+bx+c>0$ by test points.

1. All one one side
2. Solve $f(x)=0$ (This breaks $\mathbb{R}$ into intervals)
3. Test one pt in each interval.

   $+$ means >0

   $-$ means <0

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