§2.6 #s 1-4 ALL, 5, 8*, 10, 13, 17☆

*what is K in this case

$$A = Kr^2 \quad \text{for some } K \in \mathbb{R}$$

Circle: $A = \pi r^2$ $\quad K = \pi$

Area is proportional to the square of the radius.

x is jointly proportional to the cube of z and the square root of y & inversely proportional to the 5th power of w.

$$x = \frac{k z^3 \sqrt{y}}{w^5} \quad \text{for some } k.$$

Suppose $x = 1$ when $z = 2$, $y = 3$ and $w = 4$.

Model this.

$$1 = \frac{k(2)^3(\sqrt{3})}{4^5} = \frac{8\sqrt{3}}{1024}k \implies$$

$$\frac{1024}{8\sqrt{3}} = K = \frac{512}{4\sqrt{3}} = \frac{256}{2\sqrt{3}} = \frac{128}{\sqrt{3}} = \boxed{\frac{128\sqrt{3}}{3} = K}$$

$1.732 \approx \sqrt{3}$

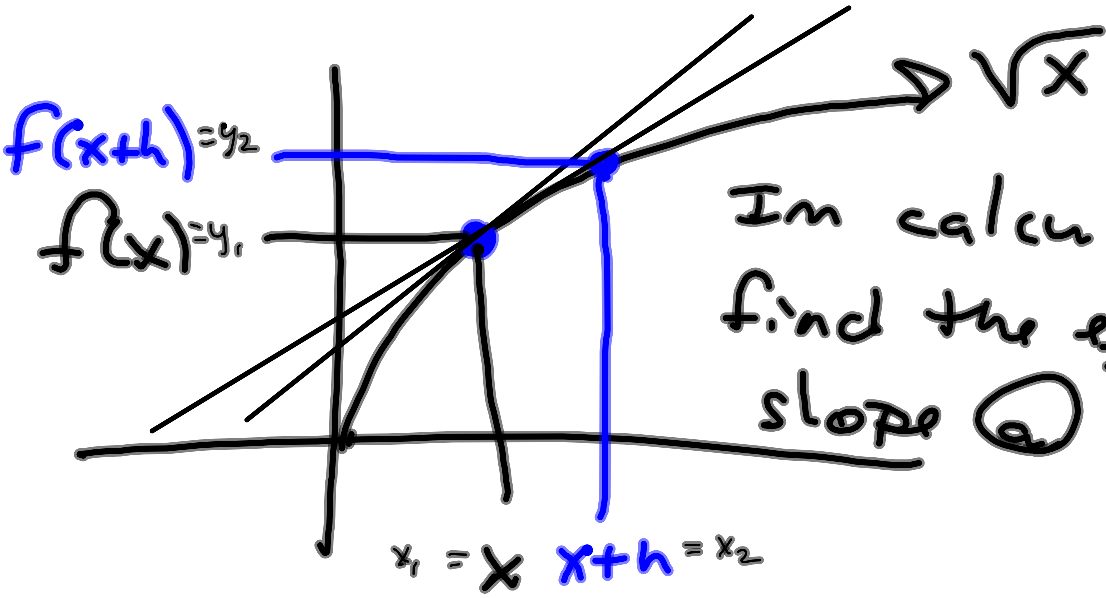
$$\frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} \quad : \quad 1.732\overline{)1.0000}$$

ugh!

$$3\overline{)1.732000} = .577333$$

One reason to rationalize the denominator

$$\frac{\sqrt{x+h} - \sqrt{x}}{h}$$ is the difference quotient.



In calculus, find the exact slope @ x

$$\frac{y_2-y_1}{x_2-x_1} = \text{Average Slope of } \sqrt{x} = \frac{f(x+h)-f(x)}{x+h-x}$$

$(a-b)$ $(a+b)$ a^2-b^2

$$= \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$ U R DONE

$h=0$ BAD!

$$= \frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \boxed{\frac{1}{\sqrt{x+h}+\sqrt{x}}}$$ STOP

and $h \to 0$ is cake $\longrightarrow \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$

Passing to the limit is Bonus

Let $f(x) = x^2 - 3x$. Simplify the difference quotient.

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h} = \frac{2xh + h^2 - 3h}{h} = \frac{h(2x + h + 3)}{h}$$

$$= 2x + h + 3$$

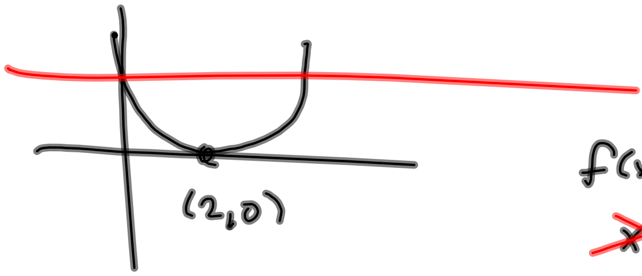
Bonus Pass to the limit and do calculus!

$\xrightarrow{h \to 0} 2x + 3$ is how steep $x^2 - 3x$ is at any x.

§2.5 #79

Find the inverse using switch and solve

$f(x) = (x-2)^2$ for $x \geq 2$



$x \geq 2$

$f(x) = x^2 - 4x + 5$

$\cancel{x =}\ y^2 - 4y + 5 = x$

$y^2 - 4y = x - 5$

$y^2 - 4y + 2^2 = x - 5 + 4$

$(y-2)^2 = x - 1$

$y - 2 = \pm\sqrt{x-1}$

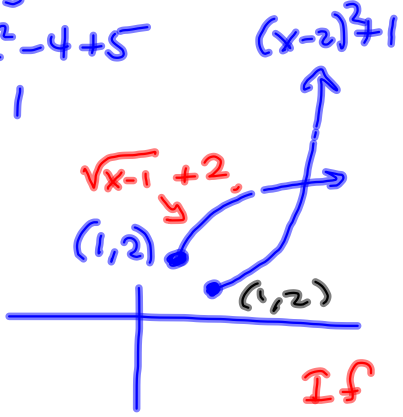
$y = 2 \pm \sqrt{x-1}$

$y = x^2 - 4x + 5$

$= x^2 - 4x + 2^2 - 4 + 5$

$= (x-2)^2 + 1$

$(h,k) = (2,1)$

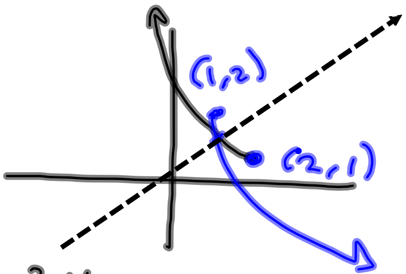


If $f(x) = (x-2)^2 + 1$ for $x \geq 2$, then

$f^{-1}(x) = \sqrt{x-1} + 2$

$= (x-2)^2+1$

$f(x) = x^2-4x+5$ for $x \le 2$



$-x+4$

$4-x$

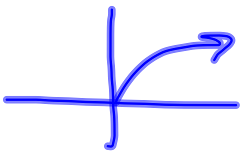
$x+y=y+x$

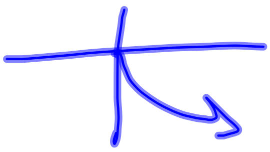
$x = y^2-4y+5$

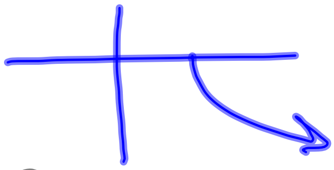
⋮

$y = \pm\sqrt{x-1}+2 = 2 \pm \sqrt{x-1}$

Take the Bottom $\frac{1}{2}$ $\quad y = -\sqrt{x-1}+2$

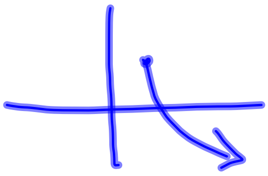






$\mathcal{D}(f) = \mathcal{R}(f^{-1})$

$(-\infty, 2]$



$-\sqrt{x-1} \quad +2$

$\mathcal{R}(f) = \mathcal{D}(f^{-1})$

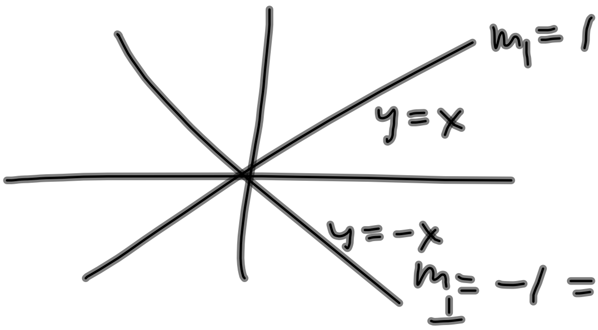
$[1, \infty)$

$$F = \frac{G m_1 m_2}{r^2}$$ G is universal gravitational constant.

Force of gravity is jointly proportional to the mass of the two bodies and inversely proportional to the square of the distance.

8 $m = \frac{K}{m_\perp}$ For $\perp$ lines, $K = -1$

slopes inversely proportional with proportionality constant $K = -1$.



$y = x$

$x = \boxed{y = x}$

$$m_\perp = -1 = \frac{-1}{1} = -\frac{1}{m_1}$$

§2.4

$f(x)=\frac{x-1}{x+2}$, $g(x)=\sqrt{x-1}$

$\mathcal{D}(f)=\{x \mid x\neq -2\}=(-\infty,-2)\cup(-2,\infty)$

$\mathcal{D}(g)=\{x \mid x-1\geq 0\}=\{x \mid x\geq -1\}=[-1,\infty)$

$(f+g)(x)=\frac{x-1}{x+2}+\sqrt{x-1}$

$\mathcal{D}=\mathcal{D}(f)\cap\mathcal{D}(g)$

$[-1,\infty)$ $(\mathcal{D}(g)\subset\mathcal{D}(f))$

$=\{x \mid x\geq -1\}$

$(fg)(x)=\left(\frac{x-1}{x+2}\right)\sqrt{x-1}$

$\left(\frac{f}{g}\right)(x)=\frac{\frac{x-1}{x+2}}{\sqrt{x-1}}$

$\mathcal{D}=$ Same as above, but $g(x)=0$ is bad.

$=(-1,\infty)$

$=\{x \mid x>-1\}$

$(f\circ g)(x)=f(g(x))$

$$=\frac{\sqrt{x-1}-1}{\sqrt{x-1}+2}$$