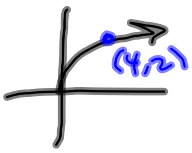
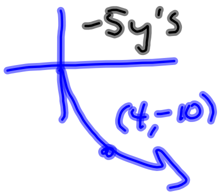
Most STUDENTS FIND it easier to do vertical stretches & put off horizontal & reflec's reflections until after horizontal shift.

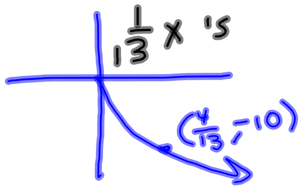
My way: $g(x) = -5\sqrt{13x-26}+5$

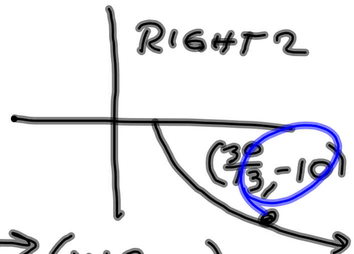
$= -5\sqrt{13(x-2)}+5$

$\sqrt{x} \longrightarrow -5\sqrt{x} \longrightarrow -5\sqrt{13x} \longrightarrow -5\sqrt{13(x-2)}$





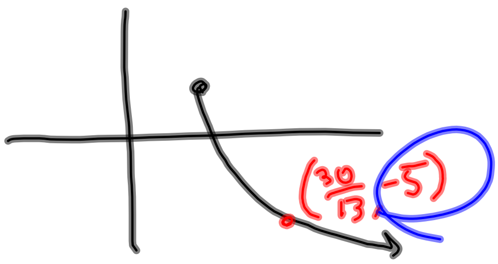




$(x,y) \longrightarrow (x+2, y)$

2 to the right $\frac{4}{13}+2 = \frac{4+26}{13} = \frac{30}{13}$

$-5\sqrt{13(x-2)}\ +5$ up 5



RECALL FROM FRIDAY 9/13

Compostion of functions.

The input becomes a function.

$(f\circ g)(x) =$ "f composed with g of x"

$= f(g(x)) = \frac{1}{g-3} = \frac{1}{\sqrt{x+2}-3}$

g's inside f!

$g(x) = \sqrt{x+2}$, $f(x) = \frac{1}{x-3}$

$\mathcal{D}(g) = \{x \mid x \geq -2\}$ $\mathcal{D}(f) = \{x \mid x \neq 3\}$

$$\boxed{\mathcal{D}(f\circ g) = \{x \mid x \in \mathcal{D}(g) \text{ AND } g(x) \in \mathcal{D}(f)\}}$$

$f(g(x))$ $= \{x \mid x \geq -2 \text{ AND } \sqrt{x+2} \neq 3\}$

$= \{x \mid x \geq -2 \text{ AND } x \neq 7\}$

Scratch

$\sqrt{x+2} = 3$

$(\sqrt{x+2})^2 = 3^2$

$x+2=9$

$\boxed{x=7}$

Recall from Friday:

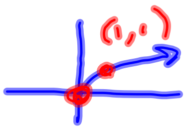
$g(x) = -5\sqrt{13x-26}+5$

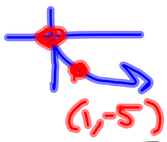
My way: $13x-26 = 13(x-2)$

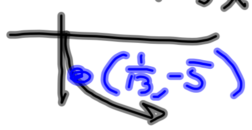
$$-5\sqrt{13(x-2)}\ +5$$

-5 y's $\quad \frac{1}{13}$ x's $\quad$ Right 2 $\quad$ up 5

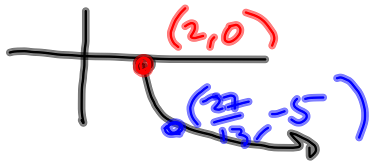
$$\sqrt{x} \longrightarrow -5\sqrt{x} \longrightarrow -5\sqrt{13x}$$

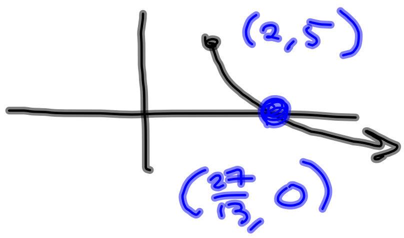






$$\longrightarrow -5\sqrt{13(x-2)} \longrightarrow -5\sqrt{13(x-2)}\ +5$$





$$\frac{1}{13}+2 = \frac{1}{13}+\frac{26}{13} = \frac{27}{13}$$

How long does it take x to get to 1?
" " " " " 13x " " " "?

§2.4 #s 1, 2, $\underbrace{15-29, 51-59, 63-69}_{\text{ODDS}}$, 105

§2.5 #s 1, 2, 3-7, 11-27, 41, 51-57 (Yes/No is OK), 67-81 ONLY ODDS.

2.5	#s 1, 2, and from here on, only odds: 3 – 7, 11 – 27, 41, 51 – 67 (A yes/no answer is OK), 67 – 81
2.6	#s 1 – 4 All, 5, 8*, 10, 13, 17 #8 – Tell me what *k* is, in this one, in addition to the question posed. #17 – Good to know what variation functions *don't* look like.

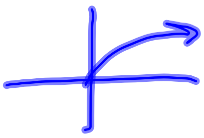
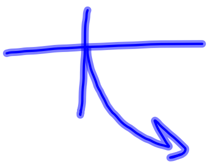
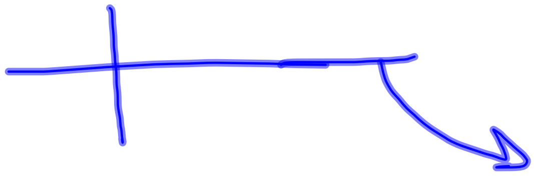
2.4 Due Wed.
2.5 Due Wed.

Students like this way:

$$\sqrt{x} \longrightarrow -5\sqrt{x} \longrightarrow -5\sqrt{x-26}$$

$$(x,y) \longrightarrow (x,-5y) \longrightarrow (x+26,-5y)$$

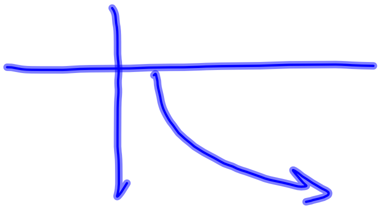
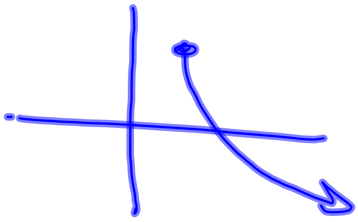
$$(1,1) \qquad (1,-5) \longrightarrow (27,-5)$$

$$\longrightarrow -5\sqrt{13x-26} \longrightarrow -5\sqrt{13x-26}+5$$

$$\left(\frac{x+26}{13}, -5y\right) \longrightarrow \left(\frac{x+26}{13}, -5y+5\right)$$

$$\left(\frac{27}{13}, -5\right) \qquad \left(\frac{27}{13}, 0\right)$$

§2.5 Fairly Light.

Invertibles & Inverses.

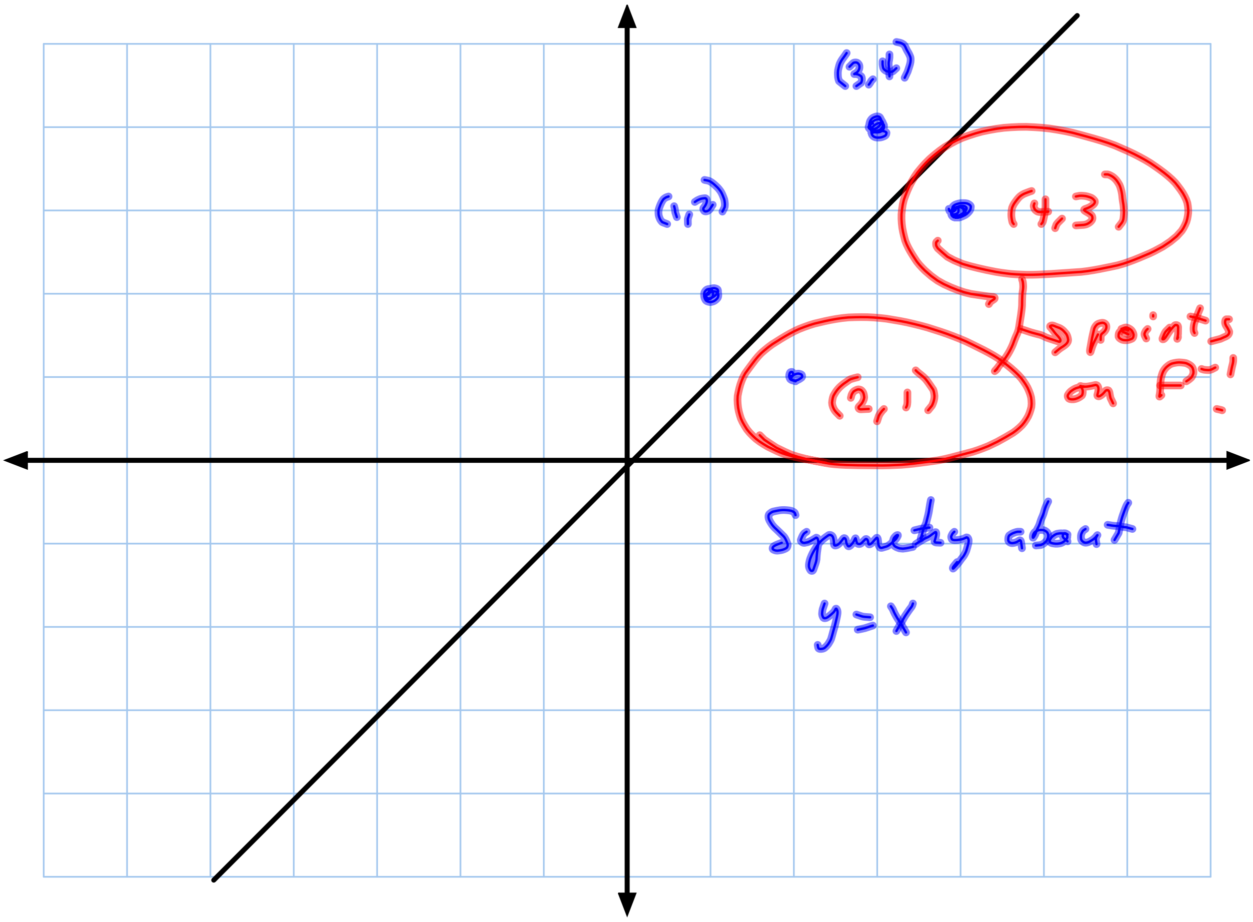
Recall Vertical Line Test.

Inverse functions involve swapping x's for y's.

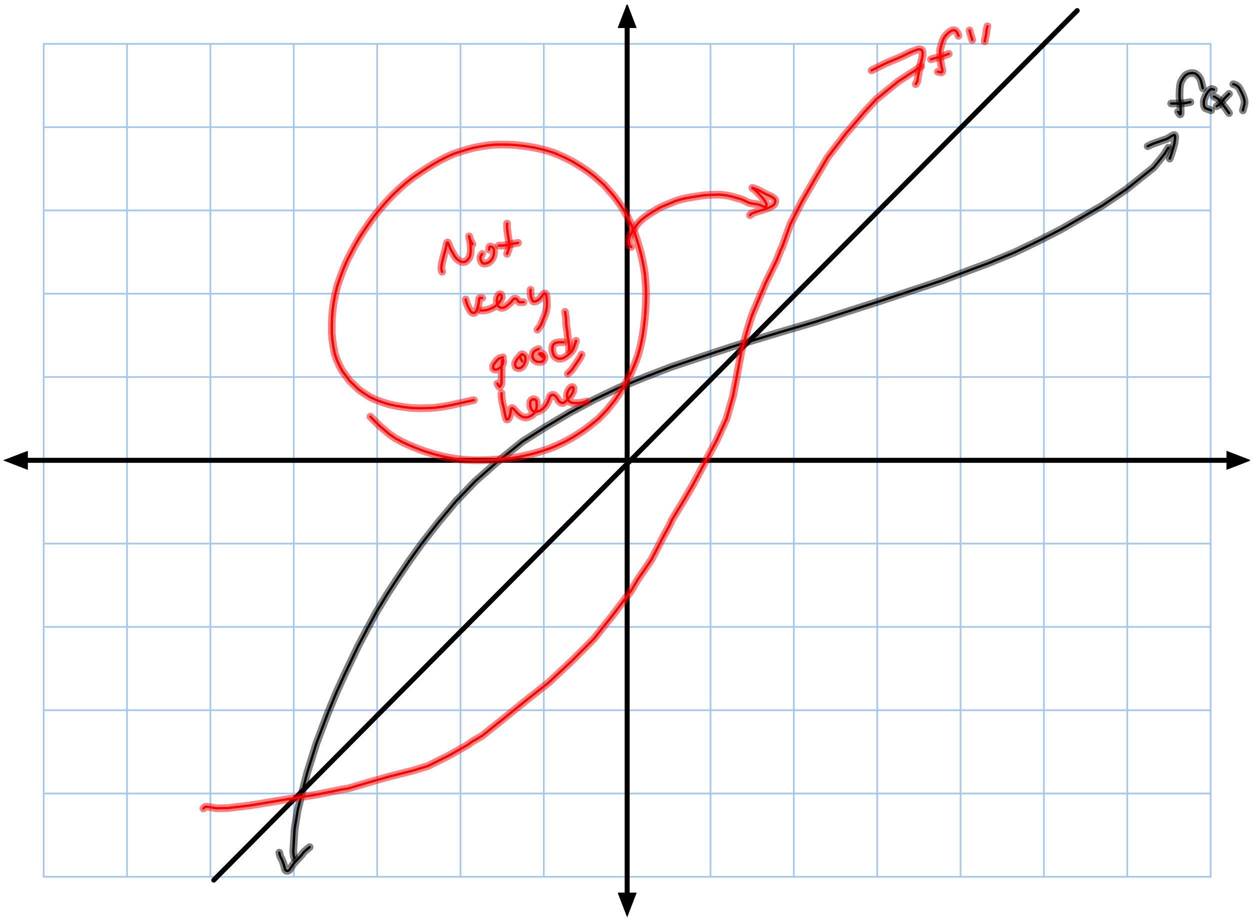
$f = \{(1,2), (3,4)\}$

$f^{-1} = \{(2,1), (4,3)\}$ For THIS to be a function, requires 1-to-1.

Sep 16-8:21 AM



(3,4)
(1,2)
(4,3)
(2,1)
points on f⁻¹
Symmetry about y = x



Sep 16-8:28 AM

2 toughies in §2.5:

① Show $f(x)$ is 1-to-1 algebraically.

② Find $f^{-1}(x)$ algebraically.

FORMAL DEF'N

$f(x)$ is 1-to-1 if $\boxed{f(x_1) = f(x_2) \Rightarrow x_1 = x_2}$

Claim: $y = 3x+2$ is 1-to-1.

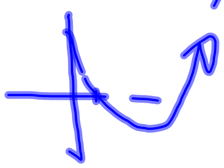
Proof: $3x_1 + 2 = 3x_2 + 2$ Solve for x_1:

$$3x_1 = 3x_2$$

$$x_1 = x_2 \quad \blacksquare$$

$y = x^2 - 3x + 2$ is NOT 1-to-1

$(x-2)(x-1)$



Show this algebraically (BONUS)

$$x_1^2 - 3x_1 + 2 = x_2^2 - 3x_2 + 2$$

$$x_1^2 - 3x_1 = x_2^2 - 3x_2$$

$$x_1^2 - x_2^2 - 3x_1 + 3x_2 = 0$$

$$(x_1 - x_2)(x_1 + x_2) - 3(x_1 - x_2) = 0$$

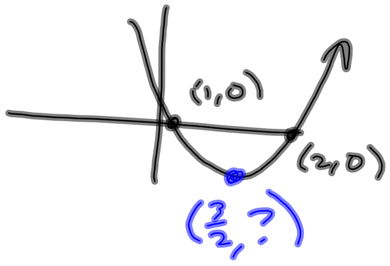
$$(x_1 - x_2)[x_1 + x_2 - 3] = 0 \Rightarrow$$

$x_1 = x_2$ OR $x_1 + x_2 - 3 = 0$

$$\Rightarrow \boxed{x_1 = 3 - x_2}$$

We CAN MAKE x^2-3x+2 1-to-1.

Restrict its domain to $\left[-\frac{3}{2}, \infty\right)$



OR to $\left(-\infty, -\frac{3}{2}\right]$
1-to-1 on ½ the picture.

<u>Find $f^{-1}(x)$</u> for $f(x) = x^2-3x+2$, restricted to $x \geq \frac{3}{2}$

$$y = x^2-3x+2$$

$$x = y^2-3y+2 \quad \text{Solve for } y.$$

$$y^2-3y+2 = x$$

$$y^2-3y \quad = x-2$$

$$y^2-3y+\left(\frac{3}{2}\right)^2 = x-2+\frac{9}{4} = x-\frac{8}{4}+\frac{9}{4} = x+\frac{1}{4}$$

$$\left(y-\frac{3}{2}\right)^2 = x+\frac{1}{4}$$

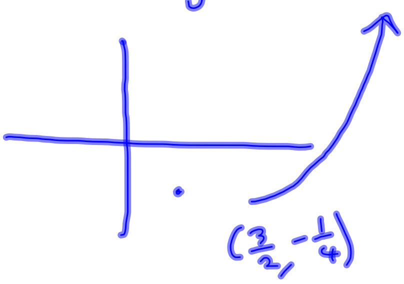
$$y-\frac{3}{2} = \pm\sqrt{x+\frac{1}{4}}$$

$$y = \frac{3}{2} \pm \sqrt{x+\frac{1}{4}} \quad \text{and}$$

$$y = \frac{3}{2} + \sqrt{x+\frac{1}{4}} \quad \text{is the one we want,}$$

$$\mathcal{D} = [3/2, \infty) = \mathcal{R}(f^{-1})$$

$$\mathcal{R} = \left[-\tfrac{1}{4}, \infty\right) = \mathcal{D}(f^{-1})$$



Show that $\frac{x-1}{x-5}$ is 1-to-1 algebraically

$$\frac{x_1-1}{x_1-5} = \frac{x_2-1}{x_2-5}$$

clearing fracs is fine if it's "="

Not fine if "≥ or <"

$$(x_1-1)(x_2-5) = (x_2-1)(x_1-5)$$

$$x_1x_2 - 5x_1 - x_2 + 5 = x_2x_1 - 5x_2 - x_1 + 5$$

$$x_1x_2 - 5x_1 - x_2 + 5 = x_2x_1 - 5x_2 - x_1 + 5$$

$$-5x_1 - x_2 = -5x_2 - x_1$$

$$-4x_1 = -4x_2$$

$$x_1 = x_2 \quad \blacksquare$$

$y = 3x + 2$ is 1-to-1. Find f^{-1}

$x = 3y + 2$

$3y + 2 = x$

$3y = x - 2$

$y = \boxed{\frac{x-2}{3} = f^{-1}(x)}$

Check:

$(f \circ f^{-1})(x) = x$

$= f\left(\frac{x-2}{3}\right) = 3\left(\frac{x-2}{3}\right) + 2$

$= x - 2 + 2$

$= x$ ✓

$f(x) = \frac{x-1}{x-5}$ find f^{-1}

$\frac{y-1}{y-5} = x$

$y - 1 = x(y-5) = xy - 5x$

$y - xy = -5x + 1$

$y(1-x) = -5x + 1$

$y = \boxed{\frac{-5x+1}{1-x} = f^{-1}(x)}$