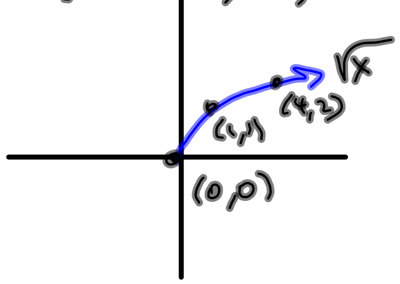
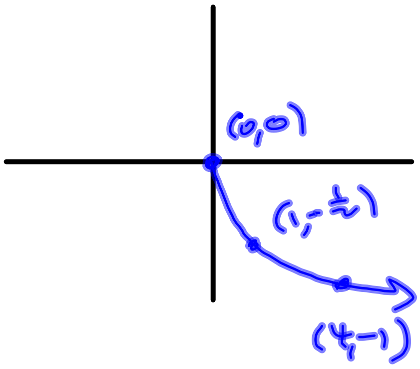
#60 $y=-\frac{1}{2}\sqrt{x+2}+4$

$y=-\sqrt{x-3}+1$

$\sqrt{x} \longrightarrow -\frac{1}{2}\sqrt{x} \longrightarrow -\frac{1}{2}\sqrt{x+2} \longrightarrow -\frac{1}{2}\sqrt{x+2}+4$

$(x,y) \mapsto (x,-\frac{1}{2}y)$





$f(x)=\sqrt{x}$

$-\frac{1}{2}f(x)=-\frac{1}{2}\sqrt{x}$

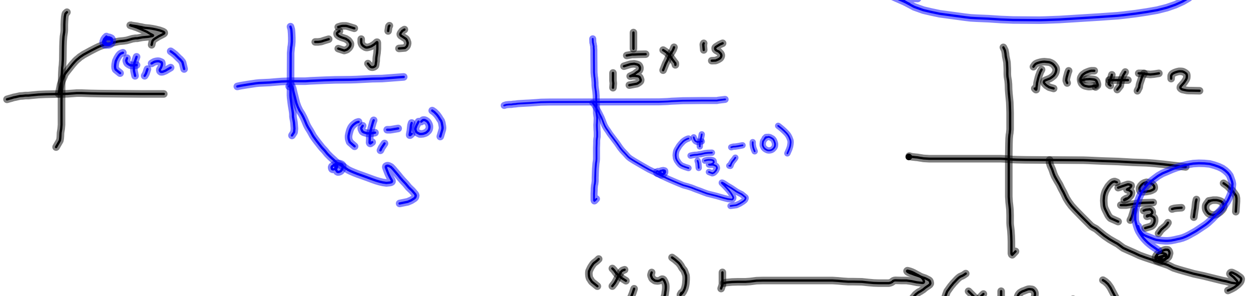
I do stretches & reflections all first.

Most STUDENTS FIND it easier to do vertical stretches & put off horizontal &reflec's reflections until after horizontal shift.

My way: $g(x) = -5\sqrt{13x-26}+5$

$= -5\sqrt{13(x-2)}+5$

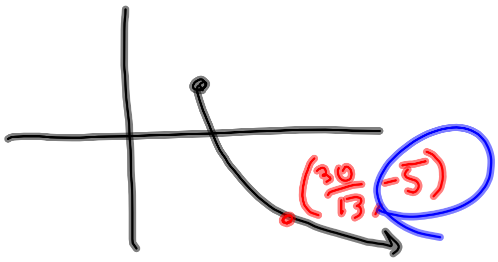
$\sqrt{x} \longrightarrow -5\sqrt{x} \longrightarrow -5\sqrt{13x} \longrightarrow -5\sqrt{13(x-2)}$



$(x,y) \longrightarrow (x+2, y)$

2 to the right $\frac{4}{13}+2 = \frac{4+26}{13} = \frac{30}{13}$

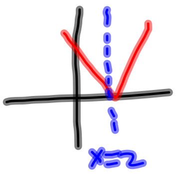
$-5\sqrt{13(x-2)} \quad +5 \quad$ up 5



2.3 67, 75

$|x-2|$ is symmetric about the line $x=2$

67 $|x-2|$ Symmetry?



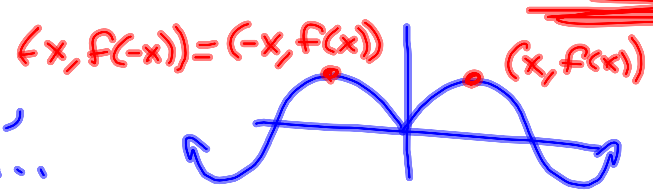
No symmetry

x-axis: Never for a function

EVEN y-axis: $f(-x)=f(x)$?

$y = c, x^2, x^4, \ldots,$

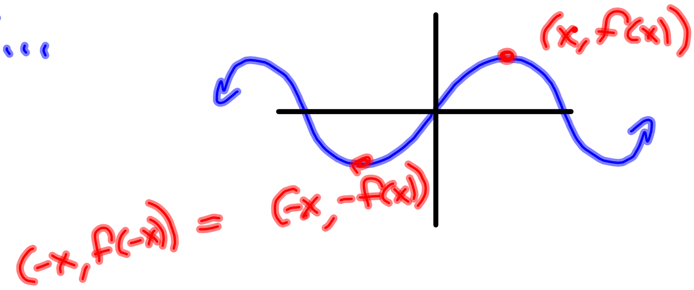
$\frac{1}{x^2}, \frac{1}{x^4}, \ldots$



ODD origin: $f(-x) = -f(x)$?

$y = x, x^3, x^5, \ldots$

$\frac{1}{x}, \frac{1}{x^3}$



ODD + ODD = ODD $\quad x + x^3$

EVEN + EVEN = EVEN $\quad 1 + x^2$

$\frac{\text{EVEN}}{\text{ODD}}$ or (EVEN)(ODD) = ODD

$\frac{\text{ODD}}{\text{ODD}}$ or (ODD)(ODD) = EVEN

Everything else: Neither.

Show that $f(x)=\dfrac{x^4+5x^2+2}{x^3-x}$ is ODD

$\boxed{\text{Pf}}$

$$f(-x)=\frac{(-x)^4+5(-x)^2+2}{(-x)^3-(-x)}=\frac{x^4+5x^2+2}{-x^3+x}$$

$$=\frac{x^4+5x^2+2}{-(x^3-x)}=-\frac{x^4+5x^2+2}{x^3-x}=-f(x)$$

ODD $\blacksquare$

$f(-x)=-f(x)$

Nice Test Question:

Show $f(x)$ is odd/even, algebraically

§2.4 Ops on funcs.

Arithmetic operations

$$(f+g)(x),\ (f-g)(x),\ (fg)(x)\ \left(\frac{f}{g}\right)(x)^{*}$$

Sum, Difference, Product, Quotient.

Are just what you expect.

$$\mathcal{D} = \mathcal{D}(f) \cap \mathcal{D}(g)$$

* except $\left(\frac{f}{g}\right)(x)$ has an additional restriction: $g(x) \neq 0$.

$f(x) = \frac{1}{x-3}$, $g(x) = \sqrt{x+2}$

$\mathcal{D}(f) = \{x \mid x-3 \neq 0\}$ $\mathcal{D}(g) = \{x \mid x+2 \geq 0\}$

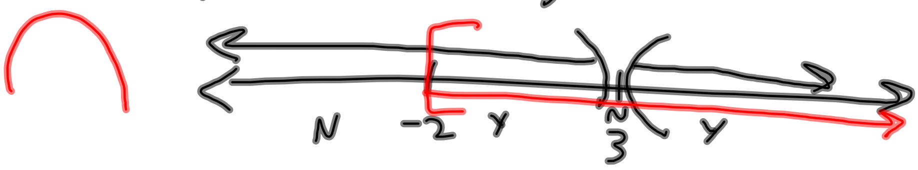
$= \{x \mid x \neq 3\}$ $= \{x \mid x \geq -2\}$

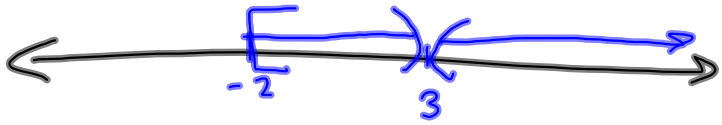
$= (-\infty, 3) \cup (3, \infty)$ $[-2, \infty)$

$(f+g)(x) = \frac{1}{x-3} + \sqrt{x+2}$

$\mathcal{D}(f+g) = \mathcal{D}(f) \cap \mathcal{D}(g)$

$\left((-\infty,3) \cup (3,\infty)\right) \cap [-2, \infty)$





$= [-2, 3) \cup (3, \infty)$

Sep 13-8:31 AM

$(f-g)(x)$ & $(fg)(x)$ are the same as $\mathcal{D}(f+g)$

What about $\left(\frac{f}{g}\right)(x)$?

$$\mathcal{D}\left(\frac{f}{g}\right) = \underline{(-2,3)\cup(3,\infty)}$$

Exactly the same, EXCEPT $g(x)=0$ is bad.

$$\boxed{\frac{\frac{1}{x+3}}{\sqrt{x+2}}} = \frac{1}{(x+3)\sqrt{x+2}}$$

Stop!

$\sqrt{x+2}=0$

$x+2=0$

$x=-2$ is bad.

Throw it out!

Compostion of functions.

The input becomes a function.

$(f \circ g)(x) =$ "f composed with g of x"

$= f(g(x)) = \frac{1}{g-3} = \frac{1}{\sqrt{x+2}-3}$

g's inside f!

$\mathcal{D}(g) = \{x \mid x \geq -2\}$ $\mathcal{D}(f) = \{x \mid x \neq 3\}$

$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ AND } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \geq -2 \text{ AND } \sqrt{x+2} \neq 3$

$= \{x \mid x \geq -2 \text{ AND } x \neq 7\}$

Scratch

$\sqrt{x+2} = 3$

$(\sqrt{x+2})^2 = 3^2$

$x+2 = 9$

$\boxed{x = 7}$

§2.4 #s 1, 2, $\underbrace{15-29, 51-59, 63-69}_{\text{ODDS}}$, 105

§2.5 #s 1-7, 11-27, maybe more to come.

Sep 13-8:57 AM