Relation – Set of ordered pairs (x, y)

Function – Relation where 1^{st} member, x is paired with only one 2^{nd} member. y

Domain – Set of x's

$\sqrt{\text{negative}}$ BAD

$\frac{\text{number}}{0}$ BAD

$\boxed{\text{Range – Set of y's}}$

Either from a graph I provide or a graph you've been trained to sketch.

Come to class Monday with
the Gallery in §2.3 Done.
~~Use~~ Leave extra space around stuff
1-side only per sheet

Finding the domain of $\sqrt{x^2-3x+2} = f(x)$ is a little advanced for us, but we actually know more about it than we might think.

Recall: Domain is generally all real numbers $= \mathbb{R} = (-\infty, \infty) = \{x \mid x \text{ is real}\}$.

There are only two exceptions that can restrict this:

(1) <u>Division by zero</u>: $\dfrac{f(x)}{g(x)}$.

Must keep $g(x) \neq 0$. Solve $g(x) = 0$ & throw it out of the domain.

(2) <u>Negative radicand</u> $\sqrt{g(x)}$

Must keep $g(x) \geq 0$. Solve $g(x) \geq 0$ & the solution <u>is</u> the domain.

So $\mathscr{D}\left(\sqrt{x^2-3x+2}\right) = \{x \mid x^2-3x+2 \geq 0\}$.

Solve $x^2-3x+2 \geq 0$

(M1) $(x-2)(x-1) \geq 0$

Sign pattern: $+ \quad - \quad +$

Yes 1 No 2 Yes

So $\mathscr{D} = (-\infty, 1] \cup [2, \infty)$.

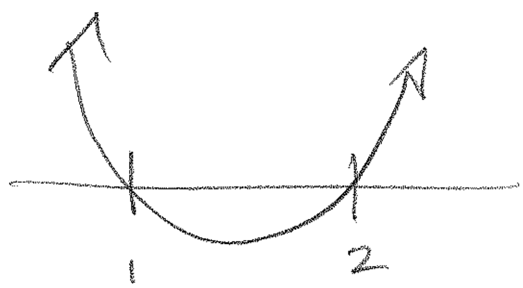
Sign pattern obtained by knowing

$g(x) = x^2 - 3x + 2$ is a parabola that

opens up. 

The factorization $(x-1)(x-2)$ tells

us $x = 1$ and $x = 2$ are the x-intercepts.



So, we're trying to decide where $x^2 - 3x + 2 \geq 0$, i.e., we're looking for intervals on which $x^2 - 3x + 2$ is AT/ABOVE the x-axis. From the picture, we get the sign pattern:

$$\longleftarrow \overset{+}{\quad} \underset{1}{|} \overset{-}{\quad} \underset{2}{|} \overset{+}{\quad} \longrightarrow$$

We want the "+" (or the zero):

$$\longleftarrow \overset{+}{\underset{\text{Yes}}{\quad}} \underset{1}{|} \overset{-}{\underset{\text{No}}{\quad}} \underset{2}{|} \overset{+}{\underset{\text{Yes}}{\quad}} \longrightarrow \implies$$

$$\boxed{(-\infty, 1] \cup [2, \infty) = \text{Domain of } \sqrt{x^2 - 3x + 2}}$$

$= \{x \mid x \leq 1 \text{ OR } x \geq 2\}$

Simplifying the difference quotient for $f(x) = 3\sqrt{x}$ is one of the tougher difference quotient simplifications

$f(x+h) = 3\sqrt{x+h} \Longrightarrow$ Difference Quotient

$$= \frac{f(x+h) - f(x)}{h} = \frac{3\sqrt{x+h} - 3\sqrt{x}}{h}$$

$$= \left(\frac{3\sqrt{x+h} - 3\sqrt{x}}{h}\right)\left(\frac{3\sqrt{x+h} + 3\sqrt{x}}{3\sqrt{x+h} + 3\sqrt{x}}\right)$$

Rationalize Numerator!

New Trick! Spinoff from $(a-b)(a+b) = a^2 - b^2$

$$= \frac{(3\sqrt{x+h})^2 - (3\sqrt{x})^2}{h(3\sqrt{x+h} + 3\sqrt{x})}$$

$$= \frac{3^2(\sqrt{x+h})^2 - 3^2(\sqrt{x})^2}{h(3\sqrt{x+h} + 3\sqrt{x})}$$

$$= \frac{9(x+h) - 9x}{h(3\sqrt{x+h} + 3\sqrt{x})} = \frac{9x + 9h - 9x}{h(3\sqrt{x+h} + 3\sqrt{x})}$$

$$= \frac{9h}{h(3\sqrt{x+h} + 3\sqrt{x})} = \boxed{\frac{9}{3\sqrt{x+h} + 3\sqrt{x}}}$$

FINAL ANSWER MAT 121

This is the form Calculus Students will need

Calculus students want to let $h \to 0$, and now it can!

Simplifying the difference quotient for $f(x) = 3\sqrt{x}$ is one of the tougher difference quotient simplifications

$f(x+h) = 3\sqrt{x+h} \implies$ Difference Quotient

$$= \frac{f(x+h) - f(x)}{h} = \frac{3\sqrt{x+h} - 3\sqrt{x}}{h}$$

$$= \left(\frac{3\sqrt{x+h} - 3\sqrt{x}}{h}\right)\left(\frac{3\sqrt{x+h} + 3\sqrt{x}}{3\sqrt{x+h} + 3\sqrt{x}}\right)$$

Rationalize Numerator!

New Trick! Spinoff from $(a-b)(a+b) = a^2 - b^2$

$$= \frac{(3\sqrt{x+h})^2 - (3\sqrt{x})^2}{h(3\sqrt{x+h} + 3\sqrt{x})}$$

$$= \frac{3^2(\sqrt{x+h})^2 - 3^2(\sqrt{x})^2}{h(3\sqrt{x+h} + 3\sqrt{x})}$$

$$= \frac{9(x+h) - 9x}{h(3\sqrt{x+h} + 3\sqrt{x})} = \frac{9x + 9h - 9x}{h(3\sqrt{x+h} + 3\sqrt{x})}$$

$$= \frac{9h}{h(3\sqrt{x+h} + 3\sqrt{x})} = \boxed{\frac{9}{3\sqrt{x+h} + 3\sqrt{x}}}$$

FINAL ANSWER MAT 121

This is the form Calculus Students will need

Calculus students want to let $h \to 0$, and now it can!