121 §4.4 #s 1-5, 9, 11, 15-21, 26, 27, 30, 41, 43, 49, 53, 59, 63, 65, 68, 75

(1) $\log_2(x) = 3$

$2^{\log_2(x)} = 2^3$

$\boxed{x = 2^3 = 8}$

$\boxed{x \in \{8\}}$

(2) $\log_3(x) = 0$

$3^{\log_3(x)} = 3^0$

$x = 1$ $\boxed{x \in \{1\}}$

(3) $\log(x+20) = 2$

$10^{\log(x+20)} = 10^2$

$x + 20 = 100$

$\boxed{x = 80}$

$x \in \{80\}$

(5) $\log(x^2 - 15) = 1$

$10^{\log(x^2-15)} = 10^1$

$x^2 - 15 = 10$

$x^2 = 25$

$\boxed{x = \pm 5}$

$\boxed{x \in \{\pm 5\}}$

(9) $-2 = \log_x(4)$

$x^{-2} = x^{\log_x(4)}$

$x^{-2} = 4$

$\frac{1}{x^2} = 4$

$1 = 4x^2 = 1$

$x^2 = \frac{1}{4}$

$x = \pm \frac{1}{2}$

$\boxed{x \in \{\frac{1}{2}\}}$

(11) $\log_x(10) = 3$

$x^{\log_x(10)} = x^3$

$10 = x^3$

$x = \sqrt[3]{10}$

$\boxed{x \in \{\sqrt[3]{10}\}}$

121 §4.4 ~~#s~~ 15-21, 26, 27, 30, 41, 43, 49, 53, 59,
63, 65, 68, 75

#s 15-24 Eq'ns involve more than one logarithm.

18 $\log_2(x+2) + \log_2(x-2) = 5$

$\log_2((x^2-4)) = 5$

$2^{\log_2(x^2-4)} = 2^5 = 32$

$x^2-4 = 32$

$x^2 = 36$

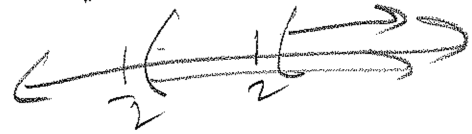
$x = \pm 6$, but $-6 \notin \mathcal{D}$

$\boxed{x \in \{6\}}$

DOMAIN

$x+2>0$ and $x-2>0$

$x>-2$ and $x>2$



Need $x>2$

$(2,\infty) = \mathcal{D}$

$\mathcal{D}$: need $\frac{x-3}{2} > 0$

and $\frac{x+2}{7} > 0$

Need $x>3$

17 $\log\left(\frac{x-3}{2}\right) + \log\left(\frac{x+2}{7}\right) = 0$

$\log\left(\frac{x^2-x-6}{14}\right) = 0$

$10^{\log\left(\frac{x^2-x-6}{14}\right)} = 10^0$

$\frac{x^2-x-6}{14} = 1$

$x^2-x-6 = 14$

$x^2-x-20 = 0$

$(x-5)(x+4) = 0$

$x = -4, 5$

$-4 \notin \mathcal{D}$

$x \in \{5\}$

121 §4.4 #s 19, 21, 26, 27, 30, 41, 49, 53, 59, 63, 65, 68, 75

(19) $\log(x+1) - \log(x) = 3$

$\log\left(\frac{x+1}{x}\right) = 3$

$10^{\log\left(\frac{x+1}{x}\right)} = 10^3$

$\frac{x+1}{x} = 1000$

$x + 1 = 1000x$

$-999x + 1 = 0$

$-999x = -1$

$\boxed{x = \frac{1}{999}}$

(21) $\log_4(x) - \log_4(x+2) = 2$

$\mathcal{D}$: $x > 0$ and $x + 2 > 0$

$x > 0$ and $x > -2$

Need $x > 0$

$\log_4\left(\frac{x}{x+2}\right) = 2$

$4^{\log_4\left(\frac{x}{x+2}\right)} = 4^2$

$\frac{x}{x+2} = 16$

$x = 16x + 32$

$-15x = 32$

$x = -\frac{32}{15}$, but $-\frac{32}{15} \notin \mathcal{D} = \{x \mid x > 0\}$

121 §4.4 #s 26, 27, 30, 41, 49, 53, 59, 63, 65, 68, 75

(26) $\log_3(x) = \log_3(2) - \log_3(x-2)$

$\mathcal{D}$: Need $x>0$ & $x-2>0$

$x>0$ & $x>2$

$\Longrightarrow x>2$

$\log_3(x) = \log_3\left(\frac{2}{x-2}\right)$

$x = \frac{2}{x-2}$ (log is 1-to-1)

$x(x-2) = x^2-2x = 2$

$x^2-2x-2=0$

$x^2-2x = 2$

$x^2-2x+1^2 = 2+1$

$(x-1)^2 = 3$

$x-1 = \pm\sqrt{3}$

$x = 1\pm\sqrt{3}$

$1-\sqrt{3} \notin \mathcal{D}$

$x \in \{1+\sqrt{3}\}$

$a=1, b=-2, c=-2$

$b^2-4ac = (-2)^2-4(1)(-2)$

$= 4+8 = 12$

$\Rightarrow \sqrt{12} = 2\sqrt{3}$

$x = \frac{2\pm2\sqrt{3}}{2} = 1\pm\sqrt{3}$

$1-\sqrt{3} \notin \mathcal{D}$ Need $x>2$

$x \in \{1+\sqrt{3}\}$

(27)

121 § 4.4 #s 27, 30, 41, 49, 53, 59, 63, 65, 68, 75

(27) $\log(4) + \log(x) = \log(5) - \log(x)$

$\log(4x) = \log\left(\frac{5}{x}\right)$ $\mathcal{D} = \{x \mid x > 0\}$

$10^{\log(4x)} = 10^{\log\left(\frac{5}{x}\right)}$

$4x = \frac{5}{x}$

$4x^2 = 5$

$x^2 = \frac{5}{4}$

$x = \pm\sqrt{\frac{5}{4}} = \pm\frac{\sqrt{5}}{2}$

$x \in \left\{\frac{\sqrt{5}}{2}\right\}$ $\left(-\frac{\sqrt{5}}{2} \notin \mathcal{D}\right)$

(30) $\log_3(x) + \log_3(1/x) = 0$

$\boxed{\mathcal{D} = \{x \mid x > 0\} = \text{Answer!}}$

$\log_3(1) = 0$

$3^{\log_3(1)} = 3^0$

$1 = 1$ Always true if $x \in \mathcal{D}$

#s 41–48 Solve. Round to 4 decimal places.

(41) $6^x = 3^{x+1}$

M1
$\log_6(6^x) = \log_6(3^{x+1})$ $a = \log_6(3)$

$x = \log_6(3)\,(x+1)$

$x = a(x+1)$ $x(1-a) = a$

$x = ax + a$ $x = \frac{a}{1-a} = \frac{\log_6(3)}{1-\log_6(3)}$

$x - ax = a$

$= \frac{\frac{\ln(3)}{\ln(6)}}{1 - \frac{\ln(3)}{\ln(6)}} \approx 1.584962501 \approx \boxed{1.5850}$

121 §4.4 #s 41, 49, 53, 59, 63, 65, 68, 75

41 $\boxed{M2}$

$\log_3(6^x) = \log_3(3^{x+1})$

$\log_3(6)\, x = x+1 \qquad a = \log_3(6)$

$ax = x+1$

$ax - x = 1$

$x(a-1) = 1$

$x = \dfrac{1}{a-1} = \dfrac{1}{\log_3(6)-1} = \dfrac{1}{\frac{\ln 6}{\ln 3} - 1} \approx 1.584962501$

$\approx \boxed{1.5850}$

$6^x = 3^{x+1}$

$\boxed{M3}$ $\ln(6^x) = \ln(3^{x+1})$

$\ln(6)\, x = \ln(3)(x+1) \qquad a = \ln 6,\ b = \ln 3$

$ax = b(x+1)$

$ax = bx + b$

$ax - bx = b$

$x(a-b) = b$

$x = \dfrac{b}{a-b} = \dfrac{\ln(3)}{\ln(6) - \ln(3)} \approx 1.584962501$

$\approx \boxed{1.5850}$

This work (3 methods) says

that

$\dfrac{\ln 3}{\ln 6 - \ln 1} = \dfrac{\frac{\ln 3}{\ln 6}}{1 - \frac{\ln 3}{\ln 6}} = \dfrac{1}{\frac{\ln 6}{\ln 3} - 1}$ weird!

121 §4.4 #s 49, 53, 59, 63, 65, 68, 75

#s 49 – 58 Same instructions

49 $e^{-\ln(w)} = 3$

$e^{\ln(w^{-1})} = 3$

$w^{-1} = 3$

$\frac{1}{w} = 3$

$\boxed{\frac{1}{3} = w}$

53 $4(1.02)^x = 3(1.03)^x$

M1 $(1.02)^x = \frac{3}{4}(1.03)^x$

$\log_{1.02}((1.02)^x) = \log_{1.02}\left(\frac{3}{4}(1.03)^x\right)$

$x = \log_{1.02}\left(\frac{3}{4}\right) + \log_{1.02}((1.03)^x)$

$x = \log_{1.02}\left(\frac{3}{4}\right) + \log_{1.02}(1.03)\, x$

$x = a + bx$, where $a = \log_{1.02}(3/4)$

$b = \log_{1.02}(1.03)$

$x - bx = a$

$x(1-b) = a$

$x = \frac{a}{1-b} = \frac{\log_{1.02}(3/4)}{1 - \log_{1.02}(1.03)}$

$= \frac{\frac{\ln(3/4)}{\ln(1.02)}}{1 - \frac{\ln(1.03)}{\ln(1.02)}} \approx 29.48717854$

$\approx \boxed{29.4872}$

M2 $\ln(4(1.02)^x) = \ln(3(1.03)^x)$

$\ln 4 + \ln(1.02)\, x = \ln 3 + \ln(1.03)\, x$

$a = \ln 4$, $b = \ln(1.02)$

$c = \ln 3$, $d = \ln(1.03)$

$a + bx = c + dx$

$bx - dx = c - a$

$x(b - d) = c - a$

$x = \frac{c-a}{b-d}$

$= \frac{\ln 3 - \ln 4}{\ln(1.02) - \ln(1.03)}$

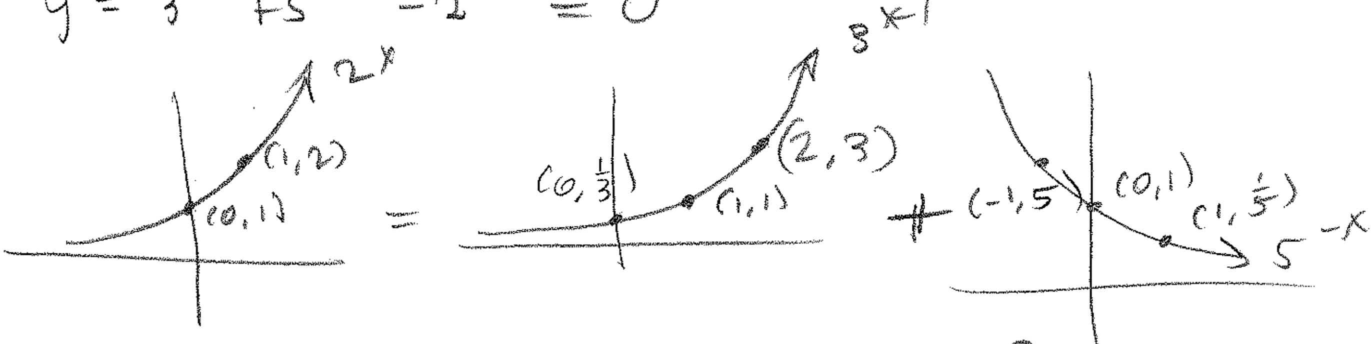
≈ 29.48717854

$\approx \boxed{29.4872}$

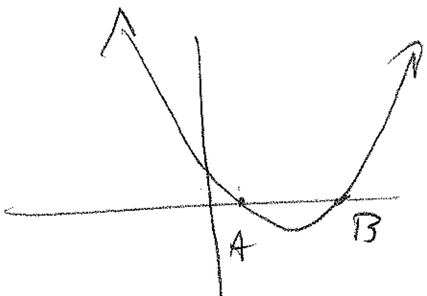
121 §4.4 #s 59, 63, 65, 68, 75

59 $2^x = 3^{x-1} + 5^{-x}$

$y = 3^{x-1} + 5^{-x} - 2^x$ $\underline{\underline{\text{SET}}}$ 0



ADD These 2

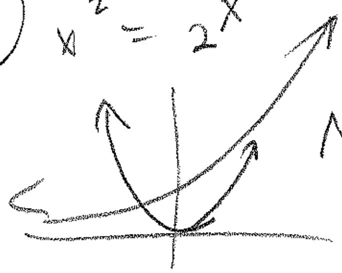


A ≈ (.19426166, 0)

B ≈ (2.7046378, 0)

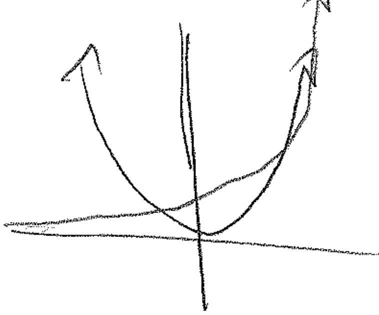
$\boxed{x \in \{.1943, 2.7046\}}$

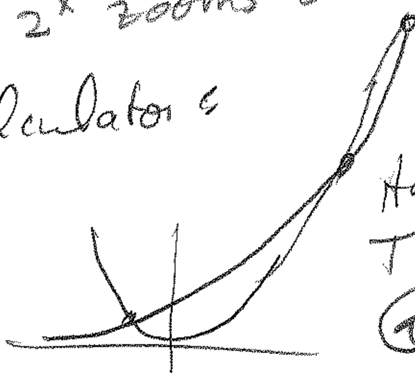
63 $x^2 = 2^x$



Not sure x^2 will catch 2^x before 2^x zooms off above it.

Graphing calculator:





Hard to draw. They're Both = 4 @ x = 2

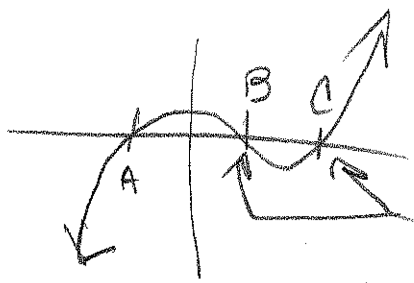
Have to zoom on relevant region to right of 2.

Graphs of 2^x & x^2 together.

Next Page: Graph $2^x - x^2$ & look for x-ints!

121 §4.4 #s 59, 63, 65, 68, 78

(63) cont'd $2^x - x^2 = f(x)$. Find zeros



The 2 crossings, here

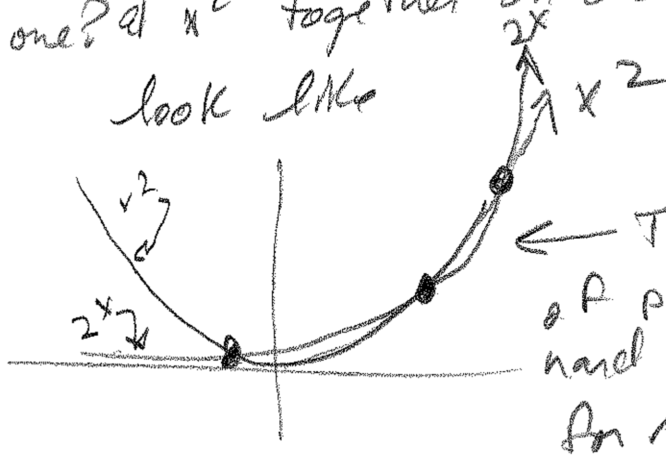
say that the graphs of 2^x & x^2 together on one graph must look like

$A \approx (-.7666647, 0)$

$B \approx (2, 0)$ The easy one?

$B = (2, 0)$

$C = (4, 0)$



This part of picture is hard to draw for me.

$2^2 - 2^2 = 0$ ✓

$2^4 - 4^2 = 0$ ✓

$16 - 16 = 0$ ✓

$2^{-.7666647} - (-.7666647)^2 \approx -7.83648 \times 10^{-9}$

$= -\dfrac{7.83648}{1{,}000{,}000{,}000} \approx 0$ ✓

$$\boxed{x \in \{-.7667, 2, 4\}}$$

121 §4.4 #s ~~62~~ 65, 68, 75

(65) $\frac{1}{2}$-life is 10,000 yrs. What's its (relative) decay rate?

$$Pe^{10000r} = \frac{1}{2}P$$

$$e^{10000r} = \frac{1}{2}$$

$$\ln(e^{10000r}) = \ln(1/2) = \ln 1 - \ln 2 = 0 - \ln 2$$
$$= -\ln 2$$

$$10000r = -\ln 2$$

$$r = -\frac{\ln 2}{10000} \approx -6.931471881 \times 10^{-5}$$

$$= \boxed{-.00006931471 81}$$

(68) 15% of C-14 has decayed. How old? 15% gone $\Longrightarrow$ 85% remains

C-14 has $\frac{1}{2}$-life of 5730 yrs

$$Pe^{5730r} = \frac{1}{2}P$$

$$e^{5730r} = \frac{1}{2}$$

$$5730r = \ln(1/2) = -\ln 2$$

$$\boxed{r = -\frac{\ln 2}{5730}}$$

So 15% gone $\rightarrow$ $Pe^{rt} = .85P$

FINAL ANSWER

$e^{rt} = .85$

$rt = \ln(.85) \longrightarrow t = \frac{\ln(.85)}{r} \approx 2344.652536$

$\approx \boxed{2345 \text{ yrs old}}$

121 §4.4 #75

75 79.3% of C-14 remains

$Pe^{rt} = .793P$

use $r = -\frac{\ln 2}{5730}$ from previous

$e^{rt} = .793$

$rt = \ln(.793)$

$t = \frac{\ln(.793)}{r} = \frac{\ln(.793)}{-\frac{\ln(2)}{5730}}$

$\approx 1917.29942\,2$

$\approx \boxed{1917 \text{ yrs ago}}$

1951 was when they dated them.

$\therefore$ 1951 − 1917 = $\boxed{34 \text{ AD}}$