121 §4.2 #s 1-8 ALL, 9, 11, 28-30 ALL, 33-45 ODD, 47, 53

(1) Inverse of an exponential is a <u>logarithm</u>

(2) Base 10 is common

(3) Base e is natural

(4) $f(x)=\log_a(x)$ is <u>increasing</u> if $a>1$ and <u>decreasing</u> if $a<1$ (Need $a>0$, too)

(5) y-axis is <u>vertical asymptote</u> for $f(x)=\log_a(x)$

(6) The <u>domain</u> of $f(x)=\log_a(x)$ is $(0,\infty)$

(7) The <u>logarithmic family</u> of funcs is all functions of the form $y=b\log_a(x-h)+k$

(8) The <u>one-to-one</u> property says that $\log_a(x)=\log_b(y)$ implies that $x=y$.

#s 9-16 Solve for the "?"

(9) $2^{?}=64=2^6 \longrightarrow \boxed{?=6}$

$2\,|\,64$, $2\,|\,32$, $2\,|\,16$, $2\,|\,8$, $2\,|\,4$, 2

(11) $3^{?}=\frac{1}{81}=\frac{1}{3^4}=3^{-4} \longrightarrow \boxed{?=-4}$

$3\,|\,81$, $3\,|\,27$, $3\,|\,9$, 3

#s 17-32 Evaluate

(28) $\log(10)=1$

(29) $\ln(e)=1$

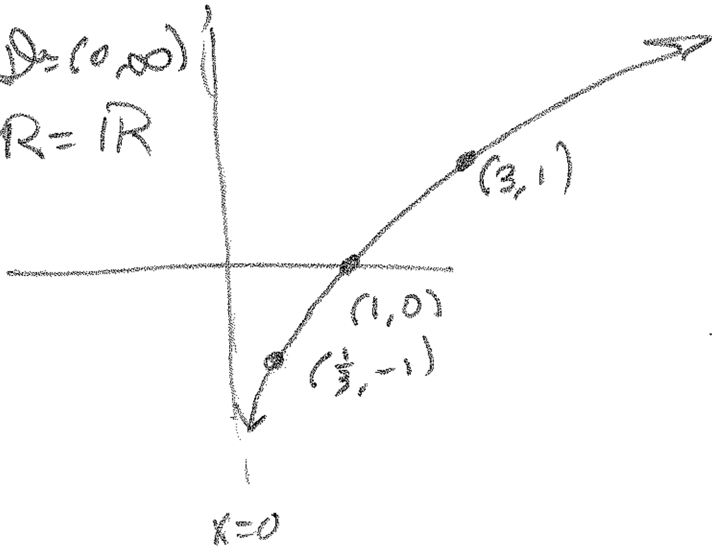
(30) $\ln(0)$ $\nexists$

121 §4.1 #s 33–43 odd, 47, 53

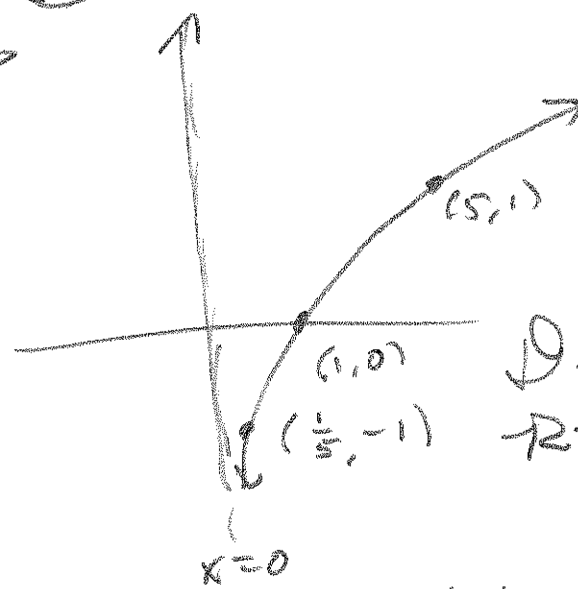
#s 33–46 sketch.

33 $y = \log_3(x)$

$\mathcal{D} = (0, \infty)$
$\mathcal{R} = \mathbb{R}$



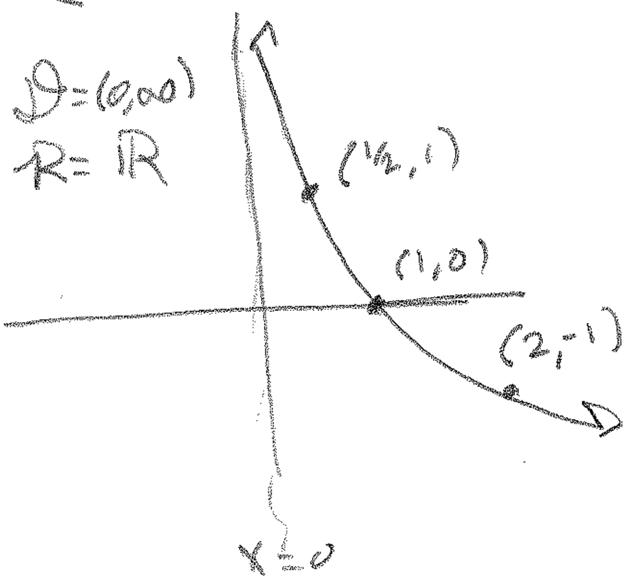
35 $f(x) = \log_5(x)$



$\mathcal{D} = (0, \infty)$
$\mathcal{R} = \mathbb{R}$

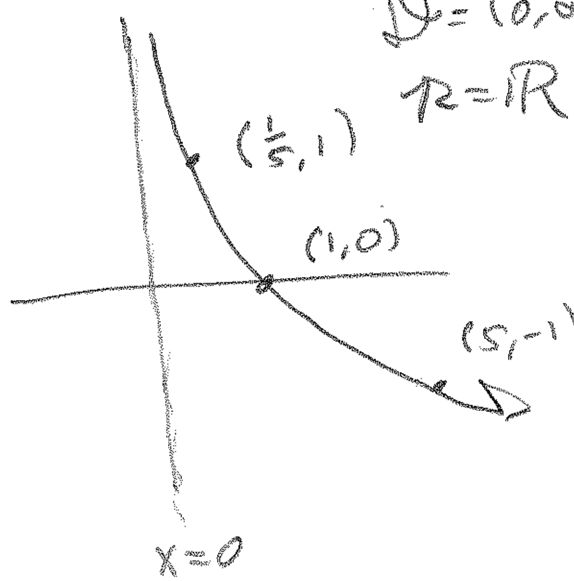
37 $y = \log_{1/2}(x)$

$\mathcal{D} = (0, \infty)$
$\mathcal{R} = \mathbb{R}$



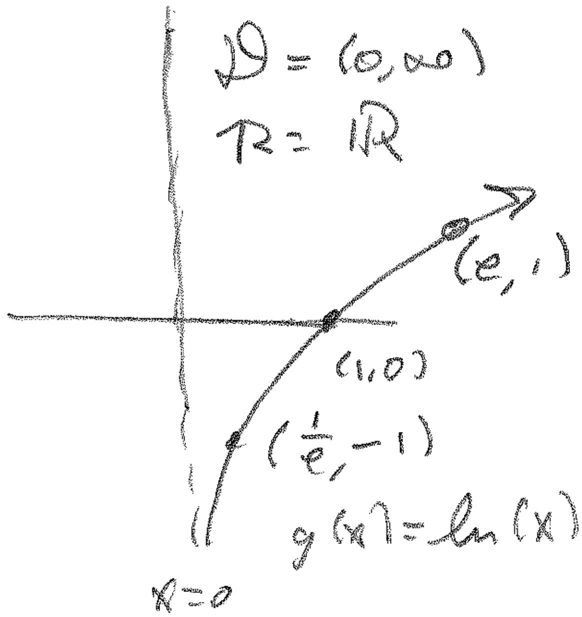
39 $h(x) = \log_{1/5}(x)$

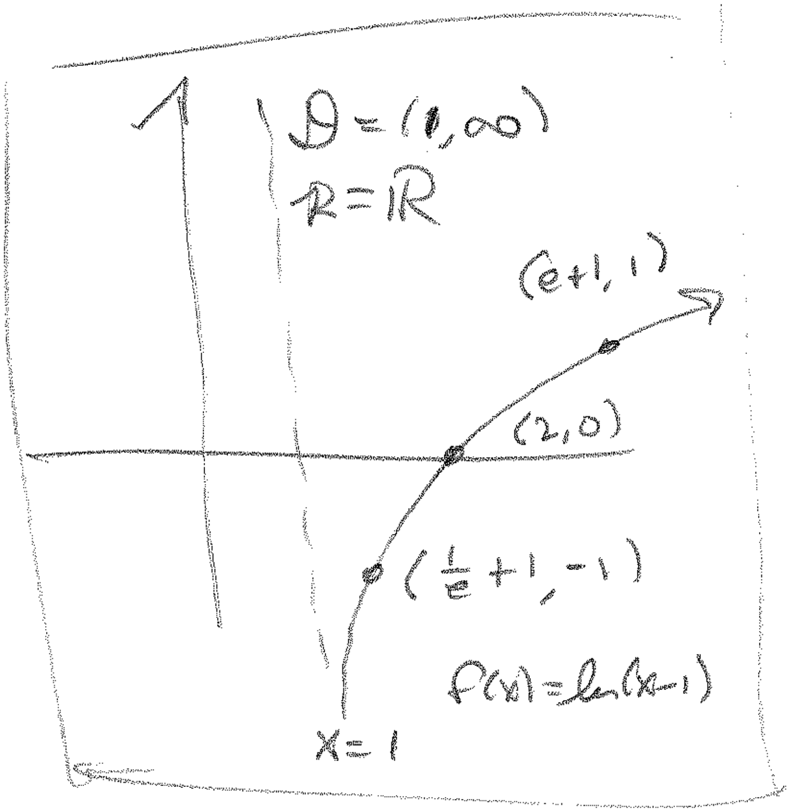
$\mathcal{D} = (0, \infty)$
$\mathcal{R} = \mathbb{R}$



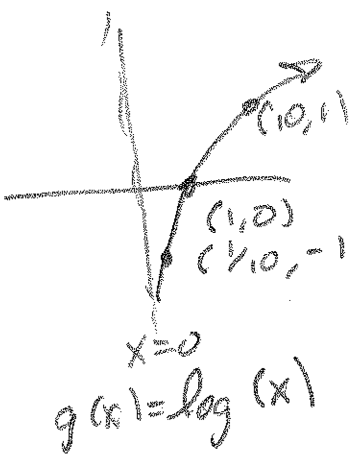
121 §4.2 #5 41–43, 47, 53

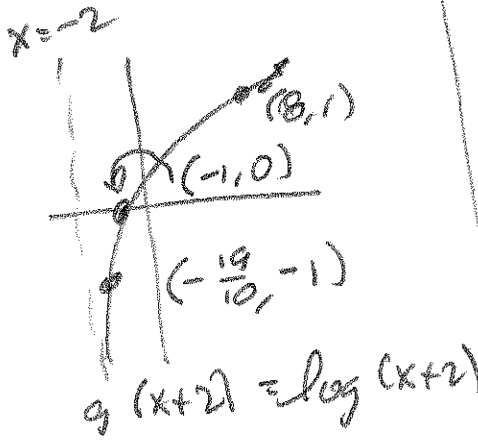
41 $f(x) = \ln(x-1)$

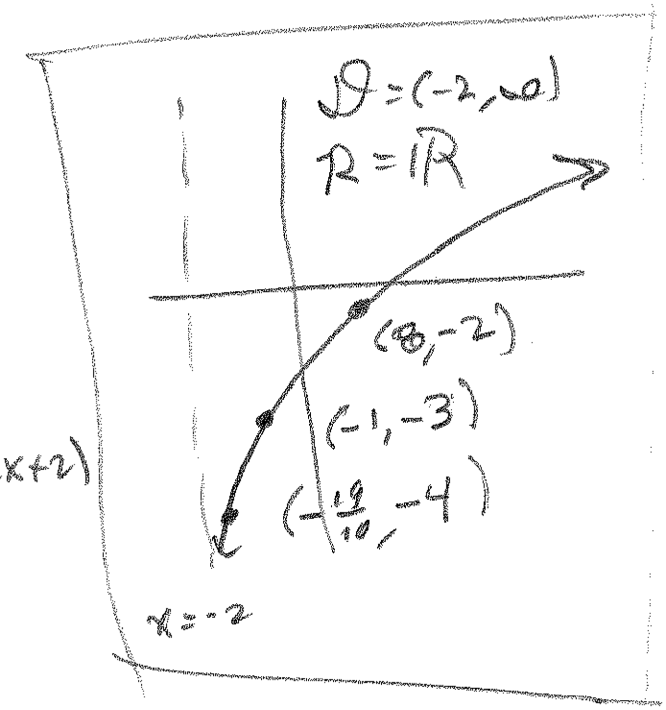




43 $f(x) = \log(x+2) - 3$





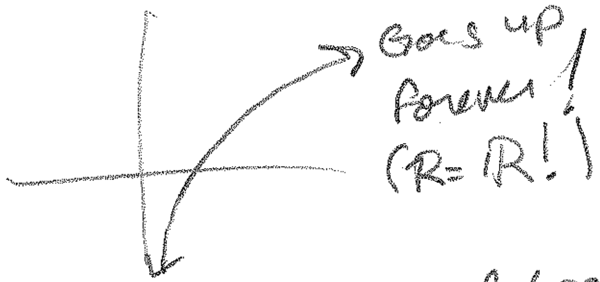


$$\frac{1}{10} - 2 = \frac{1}{10} - \frac{20}{10} = -\frac{19}{10}$$

121 §4.2 #s 47, 53

#s 47–54 Use graph / table to find the limit

47 $\lim_{x\to\infty} \log_3(x) = \infty$



$$\log_3(5000) = \frac{\ln(5000)}{\ln(3)} \approx 7.753$$

$$\log_3(10^6) = 6\log_3(10) = 6\,\frac{\ln 10}{\ln 3} \approx 12.5754$$

ADVANCED APPROACH:

PROVE It grows without bound this way:

Let M be BIG. Want to see if $\log_3(x)$ can be made bigger:

$$\log_3(x) > M \; ?$$

$$3^{\log_3(x)} > 3^M \; ?$$

$$\underline{x > 3^M}$$

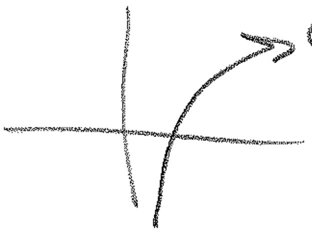
To make $\log_3(x) > 1000000000$

Take $x > 3^{100000000}$

It takes a while, but it eventually is bigger than 100000000!

121 §4.2 #s 53

53 $\lim_{x \to \infty} \log(x) = \infty$



Goes up forever is the visual

The FORMAL way is to say "Give me a big number and I can make log(x) bigger!

Want $\log(x) > BIG$

Want $10^{\log(x)} > 10^{BIG}$

Want $x > 10^{BIG}$

Now prove it by reversing the want steps:

This is a formal proof. Bonus Only.

Let M be given.

Let $x > 10^M$. Then

$\underline{\log(x)} > \log(10^M) = M\log(10) = \underline{M}$

This is a Bonus:

"PROVE $\lim_{x \to \infty} \log(x) = \infty$.