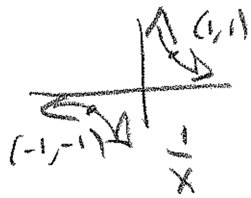
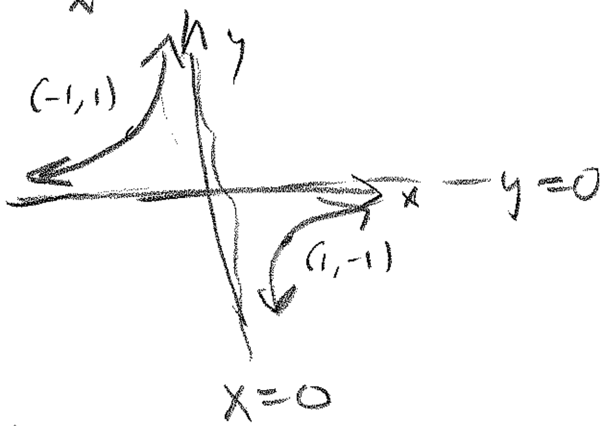
121 §3.6 I #s 33–51, 61, 71

#s 33–52 Find all asymptotes, x- and y-intercepts, and sketch

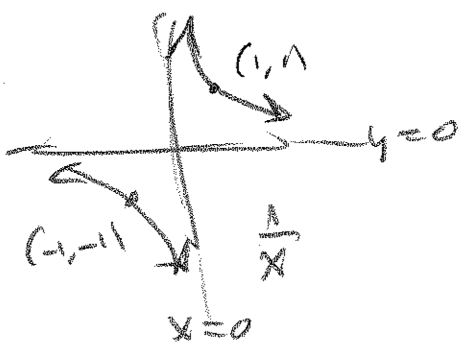
33) $f(x) = -\frac{1}{x}$ $\quad \mathcal{D} = \mathbb{R} \setminus \{0\}$

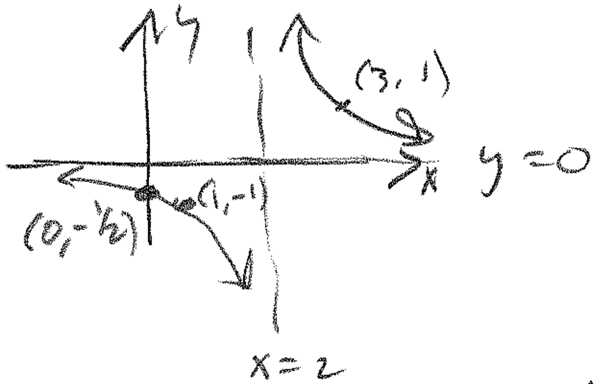


BASIC FUNC



35) $f(x) = \frac{1}{x-2}$





37) $f(x) = \frac{1}{x^2-4} = \frac{1}{(x-2)(x+2)}$

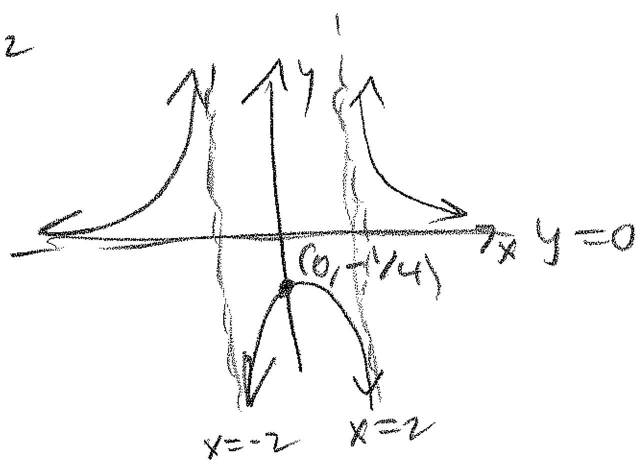
y-ints $(0, f(0)) = \boxed{(0, -\frac{1}{4})}$

$\mathcal{D} = \mathbb{R} \setminus \{\pm 2\}$ $\quad + \mid - \mid +$ (-2, 2)

$\boxed{\text{V.A.: } x = \pm 2}$

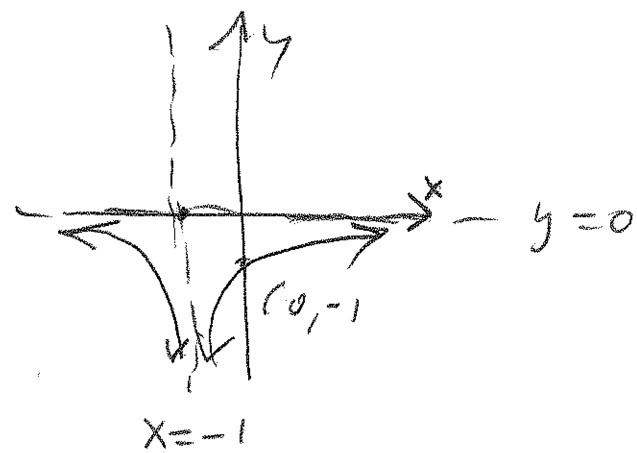
H.A.: $\frac{1}{x^2-4} \xrightarrow{x \to \infty} \frac{1}{\infty} = 0$

$\boxed{y = 0 \text{ is H.A.}}$



121 § 3.6 I #s 39-51, 61, 71

39 $f(x) = -\dfrac{1}{(x+1)^2}$



$\mathcal{D} = \mathbb{R} \setminus \{-1\}$

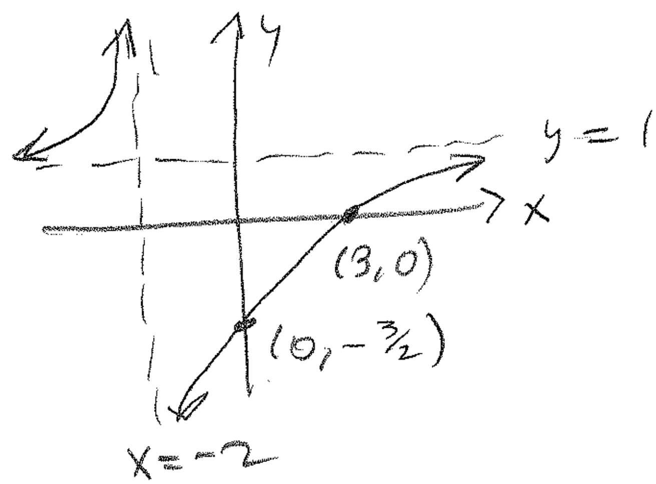
V.A.: $x = -1$

H.A.: $y = 0$ (f is "proper")

$-$ | $-$
-1

$f(0) = -1 \rightsquigarrow (0, -1)$

43 ~~41~~ $f(x) = \dfrac{x-3}{x+2}$



$\mathcal{D} = \mathbb{R} \setminus \{-2\}$

$\boxed{\text{V.A.: } x = -2}$

$\boxed{\text{H.A.: } y = 1}$ $\dfrac{x-3}{x+2} \xrightarrow{x \to \text{BIG}} \dfrac{x}{x} = 1$

$x - 3 = 0$
$x = 3 \rightsquigarrow \boxed{(3, 0)}$

$+$ | $-$ | $+$ Sign Pattern informs the graph.
-2 3

$f(0) = \dfrac{-3}{2}$

121 §3.6 I #s 43-51, 61, 71

~~43~~ 41 $f(x) = \frac{2x+1}{x-1}$

Any time degree of numerator is same as degree of denominator, just look @ highest degree terms for a horizontal asymptote.

$\frac{2x}{x} = 2 \Longrightarrow \boxed{y=2 \text{ is H.A.}}$

$\mathcal{D} = \mathbb{R} \setminus \{1\}$

$\boxed{x=1 \text{ is V.A.}}$

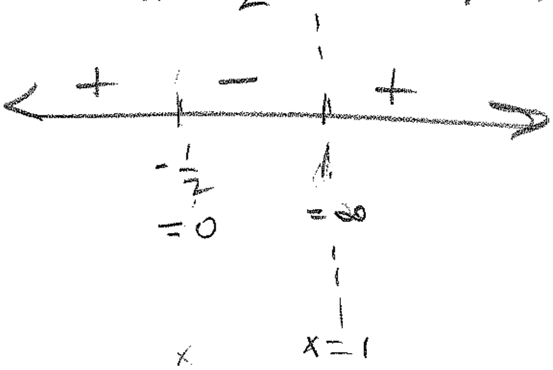
$f(0) = \frac{1}{-1} = -1 \rightsquigarrow \boxed{(0,-1)}$

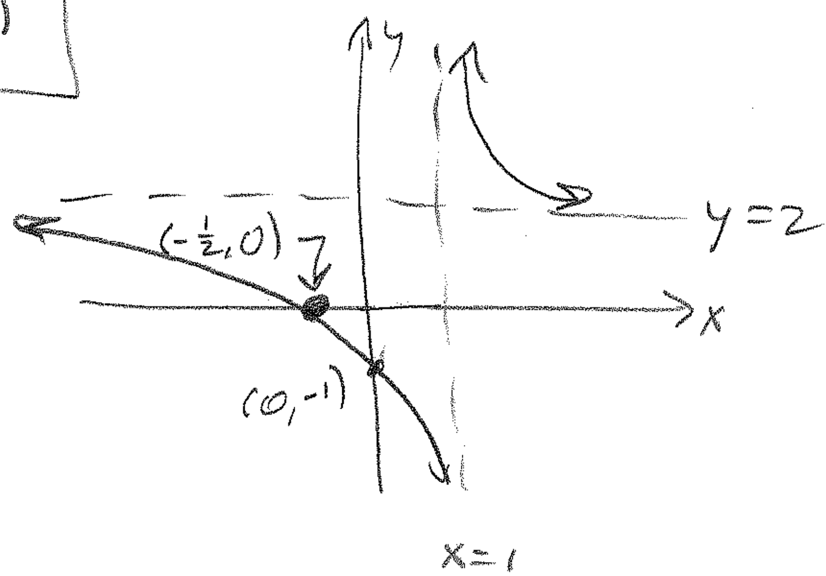
$\frac{2x+1}{x-1} = 0 \Longrightarrow$

$2x+1 = 0 \Longrightarrow$

$2x = -1 \Longrightarrow$

$x = -\frac{1}{2} \rightsquigarrow (-\frac{1}{2}, 0)$





121 §3.6 I #s 45–51, 61, 71

45 $f(x) = \dfrac{x}{x^2-1} = \dfrac{x}{(x-1)(x+1)}$

$\mathcal{D} = \mathbb{R} \setminus \{\pm 1\}$

$\boxed{x = \pm 1 \quad \text{V.A.}}$

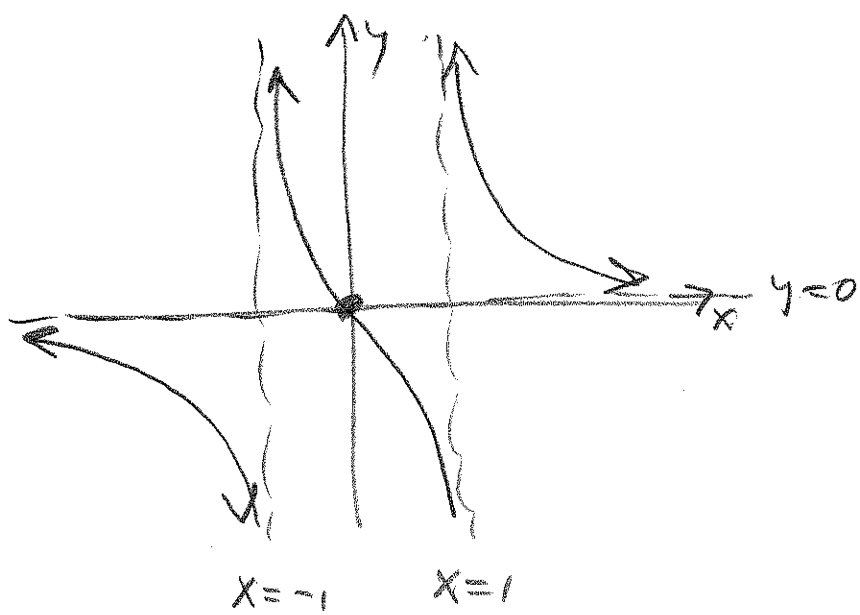
Degree of denominator is greater than degree of numerator $\Longrightarrow$ "PROPER" $\Longrightarrow$ $\boxed{y = 0 \text{ is H.A.}}$

$f(x) = 0$

$\dfrac{x}{x^2-1} = 0$

$\boxed{x = 0} \rightsquigarrow (0,0)$

$$\begin{array}{ccccccc} - & & + & & - & & + \\ & -1 & & 0 & & 1 & \end{array}$$



Might test $x=2$ to be sure the "+" on the right is correct.

$\dfrac{2}{2^2-1} = \dfrac{2}{3} > 0$ "+"

Yep.

121 §3.6 I #s 47–51, 61, 71

47 $f(x) = \dfrac{4x}{x^2-2x+1} = \dfrac{4x}{(x-1)^2}$

$\mathcal{D} = \mathbb{R} \setminus \{1\}$

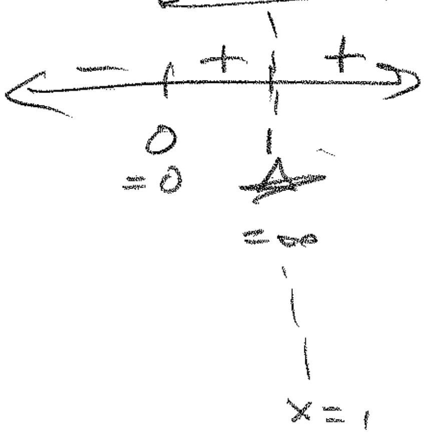
$\boxed{\text{V.A. is } x=1}$

$\boxed{\text{H.A. is } y=0}$

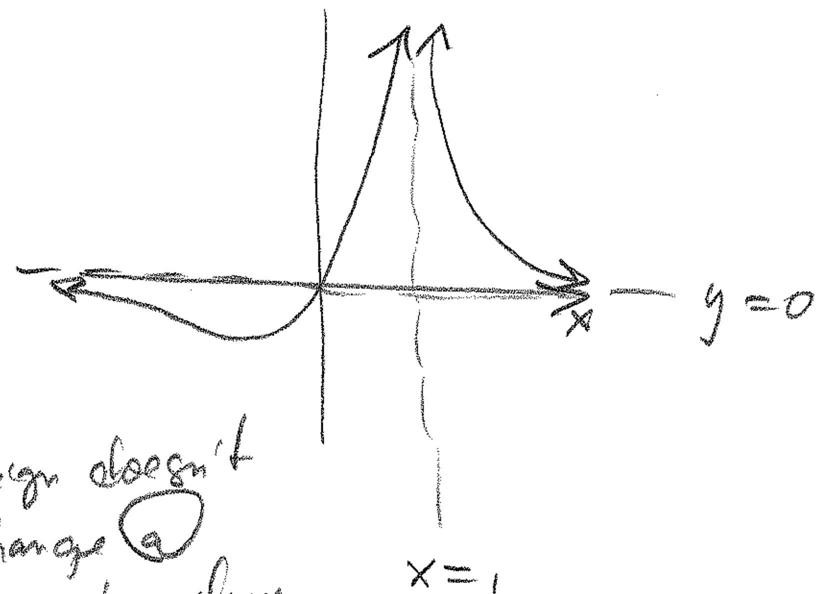
(f is "PROPER.")

$f(0) = \dfrac{0}{\ldots} = 0$

$\rightsquigarrow \boxed{(0,0)}$



Sign doesn't change @ x=1 boundary, because x=1 is root of m=2



121 §3.6 I #s 49, 51, 61, 71

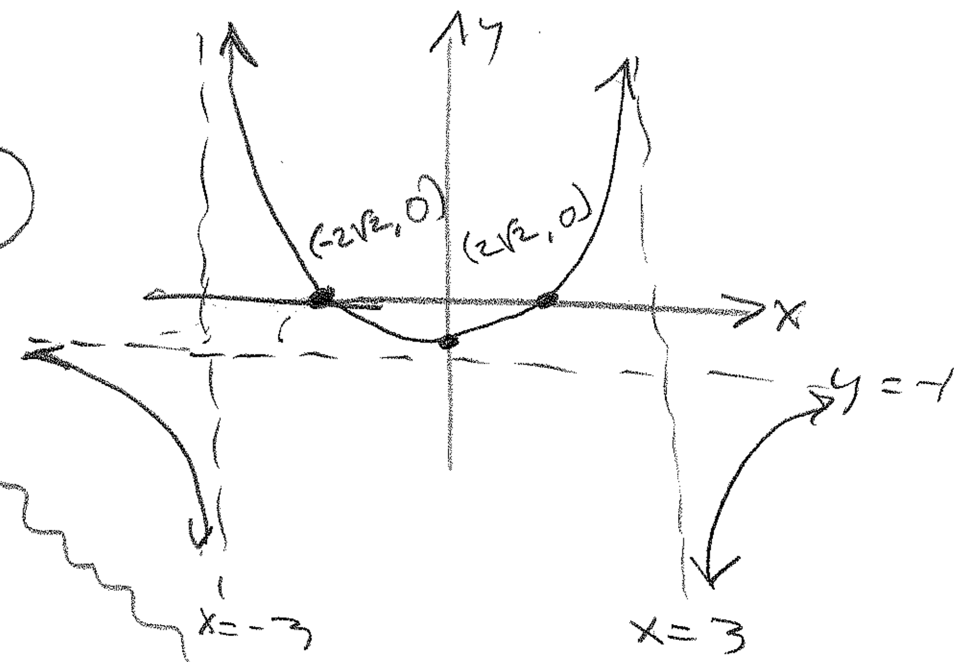
49 $f(x) = \dfrac{8-x^2}{x^2-9} = \dfrac{-x^2+8}{x^2-9} = \dfrac{-(x^2-8)}{x^2-9} = \dfrac{-(x-2\sqrt{2})(x+2\sqrt{2})}{(x-3)(x+3)}$

$\mathcal{D} = \mathbb{R} \setminus \{\pm 3\}$

$\boxed{\text{VA: } x = \pm 3}$

$\boxed{\text{H.A.:}}$ $\dfrac{-x^2}{x^2} = -1$

$\boxed{y = -1}$ BEB



$\dfrac{8-x^2}{x^2-9} = 0$

$8 - x^2 = 0$

$-x^2 = -8$

$x^2 = 8$

$x = \pm\sqrt{8}$

$x = \pm 2\sqrt{2}$

$\boxed{(2\sqrt{2}, 0),\ (-2\sqrt{2}, 0)}$

$f(0) = \dfrac{8}{-9} \rightsquigarrow \left(0, -\dfrac{8}{9}\right)$

$-$ | $+$ | $-$ | $+$ | $-$

$-3 \quad -2\sqrt{2} \quad 2\sqrt{2} \quad 3$

E

121 §3.6 I #s 51, 61, 71

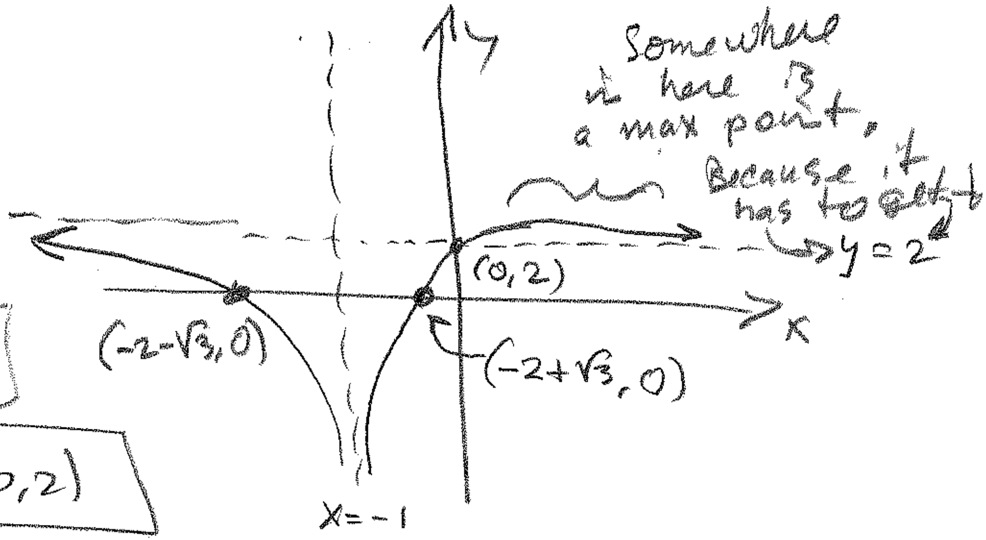
(51) $f(x) = \dfrac{2x^2+8x+2}{x^2+2x+1} = \dfrac{2(x^2+4x+2)}{(x+1)^2}$

$\mathcal{D} = \mathbb{R} \setminus \{-1\}$

$\boxed{\text{V.A. is } x=-1}$

H.A.: $\dfrac{2x^2}{x^2} = 2$

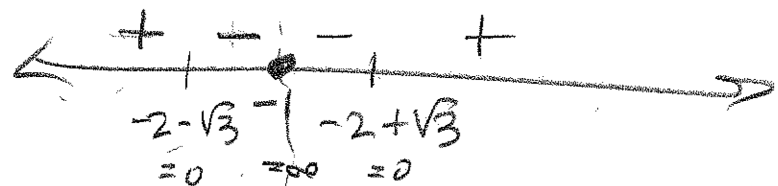
$\boxed{y=2 \text{ is H.A.}}$



$f(0) = \dfrac{2}{1} = 2 \rightsquigarrow \boxed{(0,2)}$

$f(x) = 0$:

$\dfrac{2x^2+8x+2}{(x+1)^2} = 0$



$2x^2+8x+2=0$

$x^2+4x+1=0$

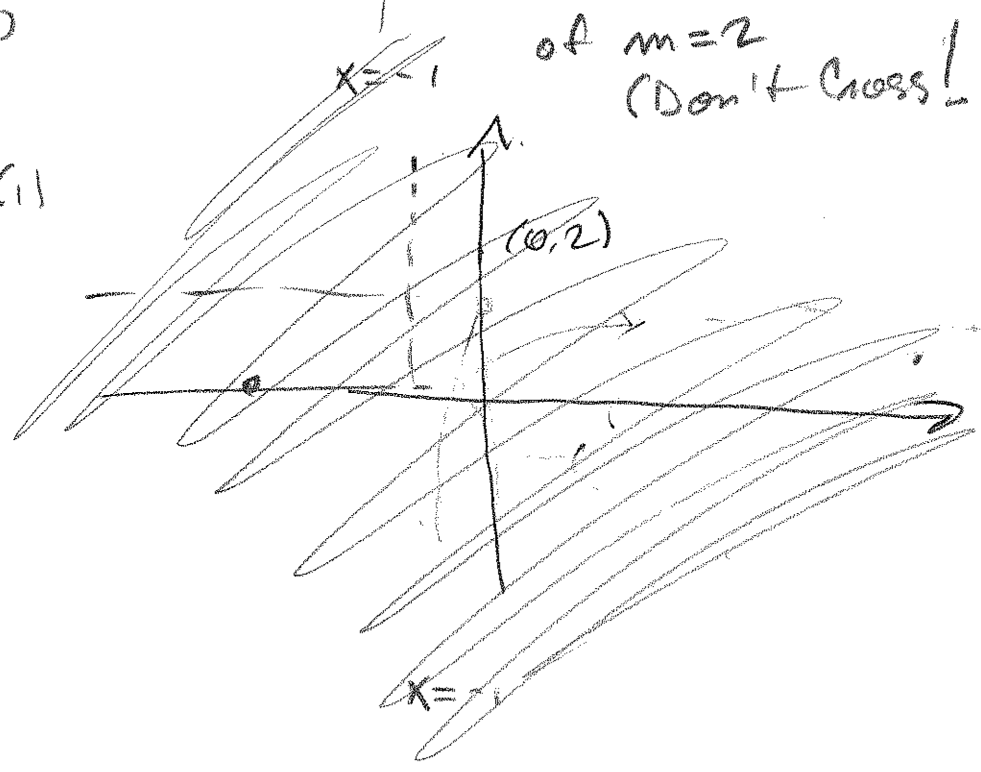
Almost missed
x = -1 is root
of m = 2
(Don't Cross!)

$a=1, b=4, c=1$

$b^2-4ac = 4^2-4(1)(1)$

$= 16-4$

$= 12$



$\sqrt{12} = 2\sqrt{3}$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{-4 \pm 2\sqrt{3}}{2}$

$= -2 \pm \sqrt{3}$

$\rightsquigarrow (-2+\sqrt{3}, 0), (-2-\sqrt{3}, 0)$

121 §3.6 I #s 61, 71

#s 61–72 Find oblique asymptote & sketch the graph of the fn.

(61) $f(x) = \dfrac{x^2+1}{x}$

Degree of numerator is GREATER than that of denominator, Use Division to find oblique Asymptote

$\mathcal{D} = \mathbb{R} \setminus \{0\}$

$\boxed{\text{V.A. is } x=0}$

H.A.: NONE!

O.A.: → O.A.

① $\begin{array}{r|l} & x \quad r\,1 \\ x & x^2+0x+1 \\ & -(x^2 \qquad\quad) \\ \hline & \qquad 1 \end{array}$

$f(x) = x + \dfrac{1}{x}$

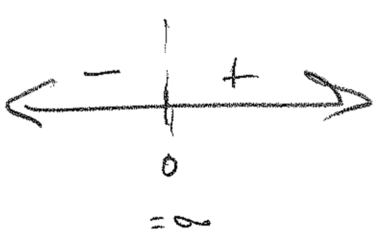
② $\dfrac{x^2+1}{x} = \dfrac{x^2}{x} + \dfrac{1}{x} = x + \dfrac{1}{x}$ → O.A.

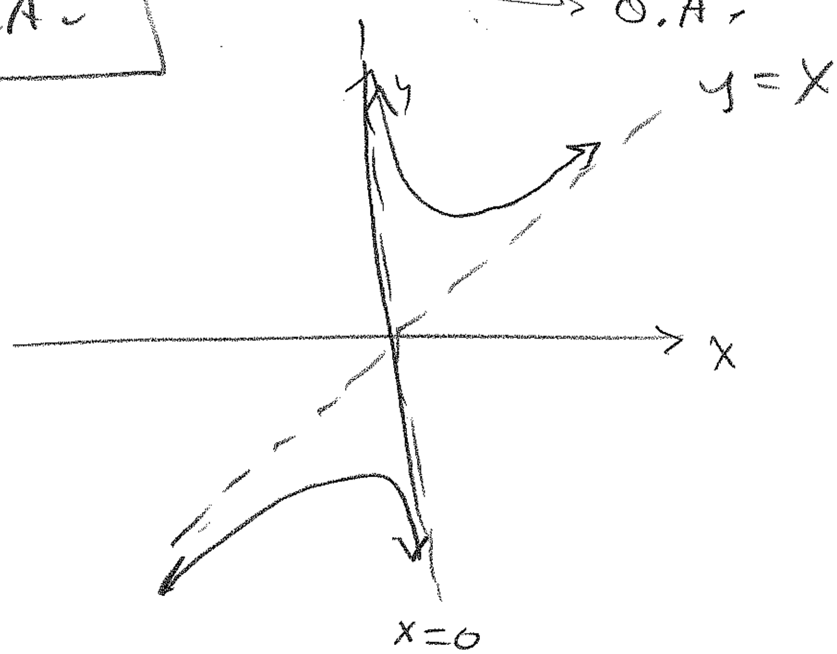
③ $\begin{array}{c|ccc} 0 & 1 & 0 & 1 \\ & & 0 & 0 \\ \hline & 1 & 0 & 1 \\ & x & c & r \end{array}$

$\dfrac{x^2+1}{x} = x + \dfrac{1}{x}$ → O.A.

UPSHOT!

$\boxed{y = x \text{ is O.A.}}$





121 §3.6 I #71

MAPLE / GRAPHING Calculator say

$f(x) = 0$ when

$x \approx 2.391382381,\ -2.164247938,\ 0.7728655577$

3 zeros, degree 3 $(-x^3 + x^2 + 5x - 4) \Longrightarrow$

All 3 are multiplicity $m = 1$.

$$f(x) \approx \frac{(x-2.39138)(x+2.16425)(x-.77287)}{(x+2)(x-1)}$$

For the oblique asymptote:

$$\begin{array}{r|l} & -x + 2 \quad r\ x \\ \hline x^2 + x - 2 & -x^3 + x^2 + 5x - 4 \\ & -(-x^3 - x^2 + 2x) \\ \hline & 2x^2 + 3x - 4 \\ & -(2x^2 + 2x - 4) \\ \hline & x \end{array}$$

Critical:

$-2.16, -2, .77, 1, 2.39$

$$\begin{array}{ccccc} -2.16 & -2 & .77 & 1 & 2.39 \end{array}$$

0

So $\dfrac{-x^3 + x^2 + 5x - 4}{x^2 + x - 2} = -x + 2 + \dfrac{x}{x^2 + x - 2}$ Tells us EB

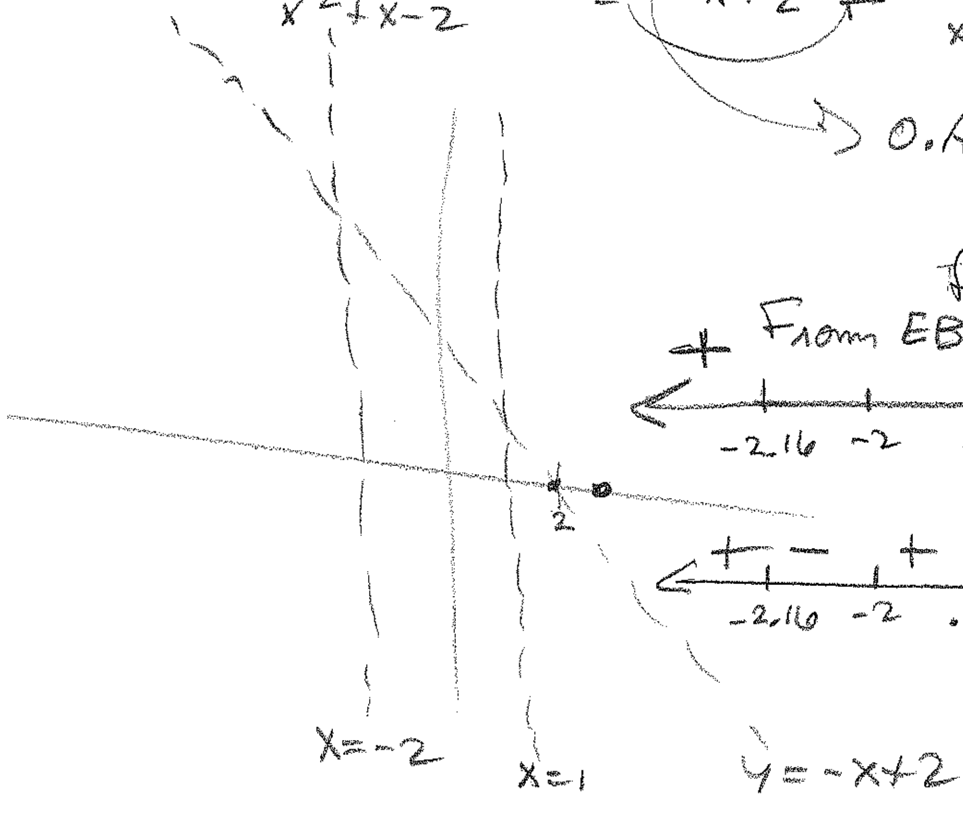
O.A. is $\boxed{y = -x + 2}$

$f(0) = \frac{-4}{-2} = 2$

From EB:

$$\begin{array}{cccccc} + & & & & & - \\ -2.16 & -2 & .77 & 1 & 2.39 & \end{array}$$

$$\begin{array}{cccccc} + & - & + & - & + & - \\ -2.16 & -2 & .77 & 1 & 2.39 & \end{array}$$



121 §3.6 I #71 F.4 13L