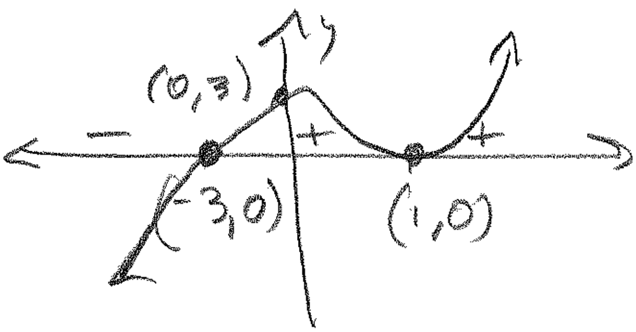
121 §3.8 #s 53-56 All, 73, 74, 85-95

#s 53-56 Make a rough sketch of the graph

53 $f(x) = (x-1)^2(x+3)$

$f(0) = (-1)^2(3) = 3 \rightsquigarrow (0,3)$

EB: $(x-1)^2(x+3) = (x^2)(x) + \text{smaller} = x^3 + \text{smaller}$



$x = 1, m = 2$ Touch

$x = 3, m = 1$ Cross

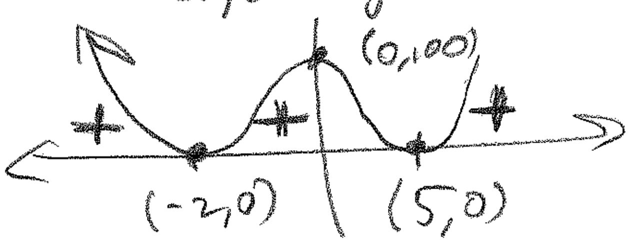
54 $f(x) = (x+2)^2(x-5)^2$

$f(0) = (2)^2(-5)^2 = 4(25) = 100 \rightsquigarrow (0,100)$

EB: $(x)^2(x)^2 = x^4$

Throw out constants.

Analyze highest power(s)



$x = 2, m = 2$

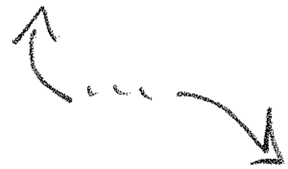
$x = 5, m = 2$

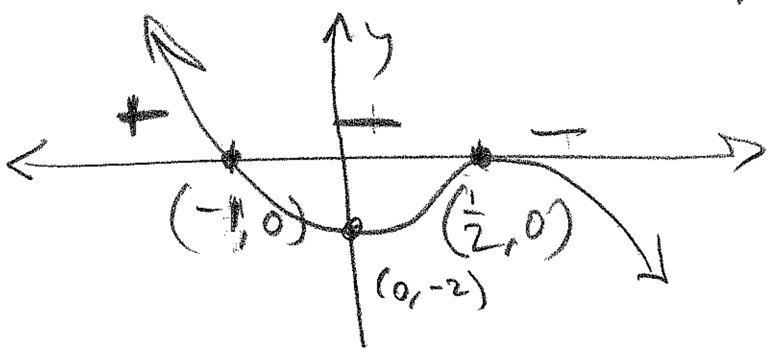
121 §3.5 #s 55, 56, 73, 74, 85–95

(55) $f(x) = -2(2x-1)^2(x+1)^3$

$f(0) = -2(-1)^2(1)^3 = -2 \rightsquigarrow (0,-2)$

E.B.: $-2(2x)^2(x)^3 = -2(4x^2)(x^3) = -8x^5$





$x = \frac{1}{2}$ $m=2$ touch

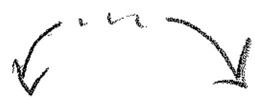
$x = -1$ $m=3$ cross

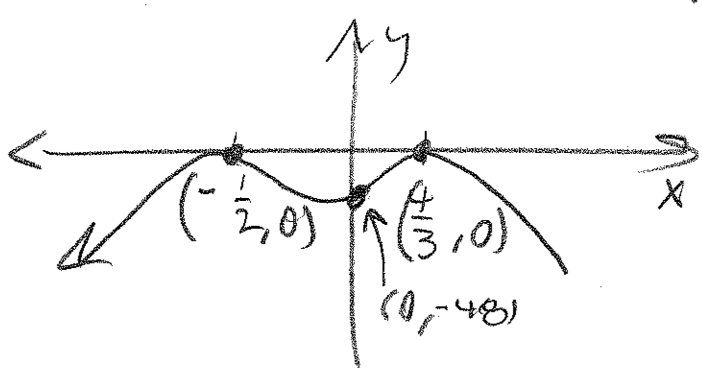
(56) $f(x) = -3(3x-4)^2(2x+1)^4$

$f(0) = -3(-4)^2(1)^4 = -3(16) = -48 \rightsquigarrow (0,-48)$

EB: $-3(3x)^2(2x)^4 = -3(9x^2)(16x^4)$

$= -3(9)(16)x^6$





$x = \frac{4}{3}$ touch $m=2$

$x = -\frac{1}{2}$ touch $m=4$

121 §3.5 #s 73, 74, 85–95

#s 65–84 Sketch the graph of each function

73) $f(x) = -x^3 - x^2 + 5x - 3$ $\pm 1, \pm 3$

$f(-x) = x^3 - x^2 - 5x - 3$

2 or 0 pos.

1 neg.

$$\begin{array}{r|rrrr} -1 & -1 & -1 & 5 & -3 \\ & & 1 & 0 & \\ \hline & -1 & 0 & \text{No} & \end{array}$$

$$\begin{array}{r|rrrr} -3 & -1 & -1 & 5 & -3 \\ & & 3 & -6 & 3 \\ \hline & -1 & 2 & -1 & 0 \end{array}$$

$(x+3)(-x^2+2x-1)$

$\boxed{x=-3, m=1 \text{ cross}}$

$-x^2+2x-1 \overset{\text{set}}{=} 0$

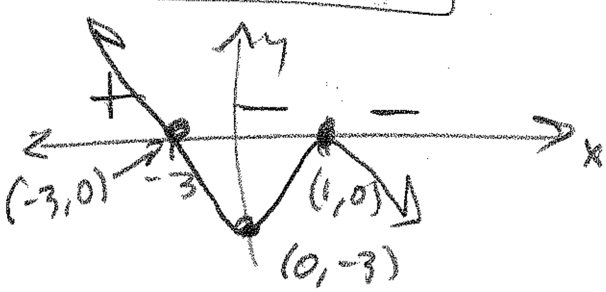
$x^2-2x+1=0$

$(x-1)^2=0$

$\boxed{x=1, m=2 \text{ touch}}$

$f(0) = -3 \rightsquigarrow \boxed{(0,-3)}$

EBS $-x^3$



121 §3.5 #s 74, 85–95

(74) $f(x) = x^3 - 10x^2 - 600x$

$= x(x^2 - 10x - 600)$

$b^2 - 4ac = (-10)^2 - 4(1)(-600)$

$= 100 + 2400 = 2500$

$\rightsquigarrow \sqrt{2500} = \sqrt{25 \cdot 100}$

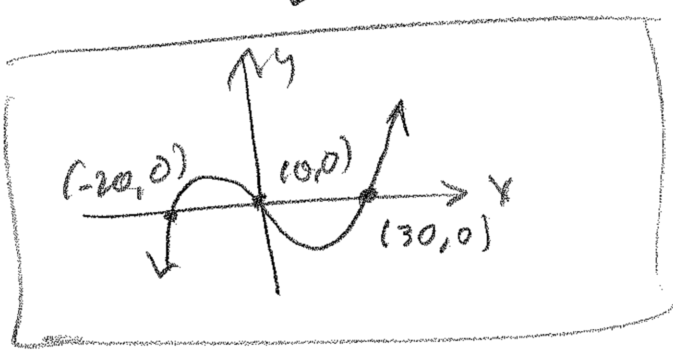
$= \sqrt{25}\sqrt{100} = 5 \cdot 10 = 50$

It factors over the rationals

$f(x) = x(x+20)(x-30)$

−	+	−	+
	−20	0	30

EB: x^3



sum	factor
$-10 = -20 + 10$	-200
$= -40 + 30$	-1200
$= -30 + 20$	-600 yes

$x^2 - 30x + 20x - 600$

$= x(x-30) + 20(x-30)$

$= (x-30)(x+20) \overset{SET}{=} 0$

$\boxed{x = -20, 30 \quad x = 0 \text{ from above}}$

121 §3.5 #s 85-95

#s 85-96 Solve each inequality

85 $x^3 - 3x > 0$

Want ">0", i.e. "+"

$x(x^2-3) > 0$

$$\boxed{x \in (-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)}$$

$x(x-\sqrt{3})(x+\sqrt{3}) > 0$

$-$ $+$ $-$ $+$

$-\sqrt{3}$ 0 $\sqrt{3}$

EB: x^3

87 $2x^2 - x^4 \le 0$

Want ≤ 0 $\le$

$x^2(2-x^2) \le 0$

$$\boxed{x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)}$$

$x^2(\sqrt{2}-x)(\sqrt{2}+x) \le 0$

(If it were "< 0" then $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$)

yes $=0$

$-$ $+$ $+$ $-$

$-\sqrt{2}$ 0 $\sqrt{2}$

EB: $-x^4$

121 §3.5 #s 89-95

89 $x^3+4x^2-x-4 > 0$

$x^2(x+4)-1(x+4) > 0$

$(x+4)(x^2-1) > 0$

$(x+4)(x-1)(x+1) > 0$

EB x^3

$-$ $+$ $-$ $+$

-4 -1 1

Want >0

$\boxed{x \in (-4,-1) \cup (1,\infty)}$

91 $x^3-4x^2-20x+48 \geq 0$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & -20 & 48 \\ & & 2 & -4 & -48 \\ \hline & 1 & -2 & -24 & 0 \end{array}$$

$x^2-2x-24=0$

$x^2-2x+1^2=24+1$

$(x-1)^2=25$

$x-1=\pm 5$

$x=1\pm 5$ → 6, -4

$x=-4, 2, 6$

$-$ $+$ $-$ $+$

-4 2 6

EB: x^3

Want ≥ 0

$\boxed{x \in [-4,2] \cup [6,\infty)}$

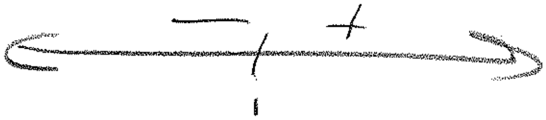
121 §3.5 #s 93, 95

(93) $x^3 - x^2 + x - 1 < 0$

$x^2(x-1) + 1(x-1) < 0$

$(x-1)(x^2+1) < 0$

x^2+1 has no real zeros,
so no role in the inequality



Want < 0:

$$\boxed{x \in (-\infty, 1)}$$

EB: x^3

(95) $x^4 - 19x^2 + 90 \le 0$

$$\begin{array}{r|rrrr} -3 & 1 & -19 & 0 & 90 \\ & & -3 & 66 & \\ \hline & 1 & -22 & & \end{array}$$

oops! Missed place holder for x^3:

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -19 & 0 & 90 \\ & & -3 & 9 & 30 & -90 \\ \hline +3 & 1 & -3 & -10 & 30 & \\ & & 3 & 0 & -30 & \\ \hline & 1 & 0 & -10 & 0 & \end{array}$$

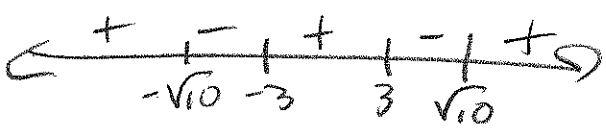
Want ≤ 0

$$\boxed{[-\sqrt{10}, -3] \cup [3, \sqrt{10}]}$$

$x^2 - 10 = 0$

$x^2 = 10$

$x = \pm\sqrt{10}$



EB: x^4