201 §3.3 IV #s 43, 47, 51, 61, 65, 67, 69, 70

#s 43–52 Descartes's rule of signs only.

(43) $f(x) = x^3+5x^2+7x+1=0$

0 sign changes $\boxed{\text{0 positive roots}}$

$f(-x) = \underbrace{-x^3+5x^2}_{1}\underbrace{-7x}_{2}\underbrace{+1}_{3}$

$\boxed{\text{3 or 1 negative roots}}$

(47) $f(y) = y^4+5y^2+7$

$\boxed{\text{0 positive}}$

$f(-y) = y^4+5y^2+7$

$\boxed{\text{0 negative}}$

(4 nonreal.)

(51) $x^5+x^3+5x=0$

0 positive

$f(-x) = -x^5-x^3-5x$

$\boxed{\text{0 negative}}$

(4 nonreal!!)

$x=0$ is the only real root

For fun:

$u=y^2$

$u^2+5u+7=0$

$u^2+5u \quad =-7$

$u^2+5u+\left(\frac{5}{2}\right)^2=-7+\frac{25}{4}$

$\left(u+\frac{5}{2}\right)^2=-\frac{3}{4}$

$u+\frac{5}{2}=\pm\sqrt{-\frac{3}{4}}=\pm\frac{i\sqrt{3}}{2}$

$u=\frac{-5\pm i\sqrt{3}}{2}=y^2$

To find y, we'd have to have some notion of

$\pm\sqrt{\frac{-5\pm i\sqrt{3}}{2}}$, which we don't, because we're not taking <u>Complex</u> <u>Analysis</u>.

121 §3.3 II #s 61, 65, 67, 69, 70

#s 53–60 Use Thm on bds to establish best integral bounds for the roots (real) of eq'n.

#s 61–76 Use thms on roots to find all real & imaginary roots

(61) $x^3-4x^2-7x+10=0$

Descartes's: 2 or 0 pos.

$f(-x)=-x^3-4x^2+7x+10$

one neg.

p's: 10

q's: 1

$\frac{p}{q}$'s: $\pm1, \pm2, \pm5, \pm10$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -7 & 10 \\ & & 1 & -3 & -10 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x=-2,\ x=5$

zeros: $x=-2, 1, 5$
each multiplicity $=1$

* you can always clobber the quadratic w/ formula or completing the square.

$\therefore$ $f(x)=(x-1)(x+2)(x-5)$
see sketch @ end

201 §3.3 #s 65, 67, 69, 70

(65) $f(x) = x^4 + 2x^3 - 7x^2 + 2x - 8 \overset{set}{=} 0$

p's: 8 $\quad \frac{p}{q}$'s: $\pm 1, \pm 2, \pm 4, \pm 8$

q's: 1

Descarte's:

3 or 1 pos.

$f(-x) = \underbrace{x^4 - 2x^3}_{1} \underbrace{- 7x^2 - 2x}_{2} \underbrace{- 8}_{3}$

3 or 1 neg.

$$\begin{array}{r|rrrrr} 2 & 1 & 2 & -7 & 2 & -8 \\ & & 2 & 8 & 2 & 8 \\ \hline -4 & 1 & 4 & 1 & 4 & 0 \\ & & -4 & 0 & -4 & \\ \hline & 1 & 0 & 1 & 0 & \end{array}$$

$x^2 + 1 = 0$

$x^2 = -1$

$x = \pm\sqrt{-1} = \pm i$

$\boxed{\text{zeros: } 2, -4, \pm i \quad m = 1\ \forall.}$

$\boxed{f(x) = (x-2)(x+4)(x-i)(x+i)}$ See sketch @ end

(67) $6x^3 + 25x^2 - 24x + 5 = 0$

p's: 5 $\quad \frac{p}{q}$'s: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$

q's: 6 $\quad \pm 5, \pm \frac{5}{2}, \pm \frac{5}{3}, \pm \frac{5}{6}$

$$\begin{array}{r|rrrr} -5 & 6 & 25 & -24 & 5 \\ & & -30 & 25 & -5 \\ \hline & 6 & -5 & 1 & 0 \end{array}$$

$6x^2 - 5x + 1 = 0$

$a = 6, b = -5, c = 1$

$b^2 - 4ac = (-5)^2 - 4(6)(1)$

$= 25 - 24 = 1 \rightsquigarrow \sqrt{1} = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{5 \pm 1}{2(6)} = \frac{5 \pm 1}{12}$ $\quad \frac{6}{12} = \frac{1}{2}$, $\quad \frac{4}{12} = \frac{1}{3}$

$\boxed{\text{zeros: } \frac{1}{3}, \frac{1}{2}, -5 \quad m = 1\ \forall.}$

$f(x) = 6(x-5)(x-\frac{1}{3})(x-\frac{1}{2})$

$\boxed{= (x+5)(3x-1)(2x-1) \text{ for style pts.}}$

121 § 3.3 II #'s 69, 70

(69) $x^4+2x^3-3x^2-4x+4 = f(x) \overset{\text{set}}{=} 0$

p's: 4 $\quad \frac{p}{q}$'s: $\pm 1, \pm 2, \pm 4$

q's: 1

Descartes: 2 or 0 pos

$f(-x) = x^4-2x^3-3x^2+4x+4$

2 or 0 neg.

$$\begin{array}{r|rrrrr}
1 & 1 & 2 & -3 & -4 & 4 \\
  &   & 1 & 3 & 0 & -4 \\
\hline
1 & 1 & 3 & 0 & -4 & 0 \\
  &   & 1 & 4 & 4 & \\
\hline
  & 1 & 4 & 4 & 0 &
\end{array}$$

$(x-1)(x^3+3x^2-4)$

$(x-1)^2(x^2+4x+4)$

$x^2+4x+4 = 0$

$(x+2)^2 = 0$

$x+2 = \pm 0$

$x = -2, m = 2$

$\boxed{(x-1)^2(x+2)^2}$

see sketch @ end.

$\boxed{\text{zeros: } x=-1,\ x=2 \quad m=2\ \forall.}$

121 §3.3 IV #70

(70) $x^5+3x^3+2x = f(x) \overset{\text{set}}{=} 0$

Takes some doing!

$x(x^4+3x^2+2) = 0$

$\boxed{x=0}$ OR $x^4+3x^2+2=0$

Our work will show that $\pm 1, \pm 2$ from Rational Zeros Theorem is no help.

What, then? RECOGNIZE

x^4+3x^2+2 is "Quadratic in form."

Let $u=x^2$, then

$u^2+3u+2=0$

$(u+2)(u+1)=0 \Rightarrow$

$u=-2$ OR $u=-1$

$x^2=-2$

$x=\pm\sqrt{-2} = \boxed{\pm i\sqrt{2}}$

$x^2=-1$

$x=\pm\sqrt{-1}$

$= \boxed{\pm i}$

Zeros:

$x = 0, \pm i\sqrt{2}, \pm i$

$f(x)$ factors as $\boxed{x\cdot(x-i)(x+i)(x-i\sqrt{2})(x+i\sqrt{2})}$

See Sketches

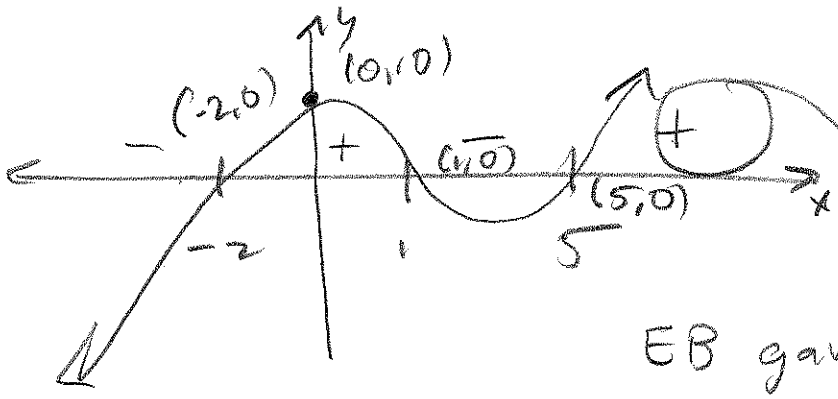
121 §3.3 IV Sketches

Homework didn't ask for factored form, but I did it, anyway, to show you the relationship between zeros (roots) and factors.

#61 $f(x) = (x-1)(x+2)(x-5) = x^3 - 4x^2 - 7x + 10$

$f(0) = 10 \rightsquigarrow (0,10)$

E.B.: x^3

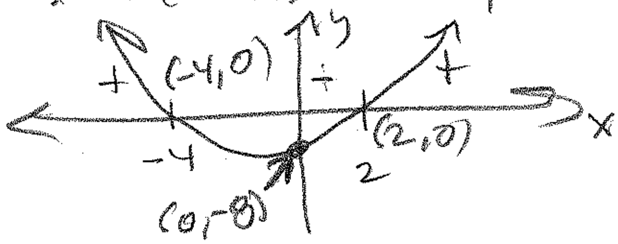


rough sketch

EB gave me . The rest from alternating signs

#65 $f(x) = x^4 + 2x^3 - 7x^2 + 2x - 8 = (x-2)(x+4)(x-i)(x+i)$

$x = \pm i$ has no expression in the graph

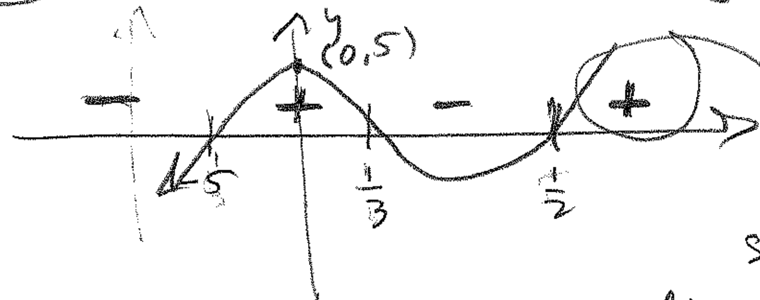


$f(0) = -8 \rightsquigarrow (0,-8)$

x^4

121 §3.3 IV Extra

(67) $f(x) = 6x^3 + 25x^2 - 24x + 5 = 6(x-5)(x-\frac{1}{3})(x-\frac{1}{2})$



$f(0) = 5 \rightsquigarrow (0,5)$

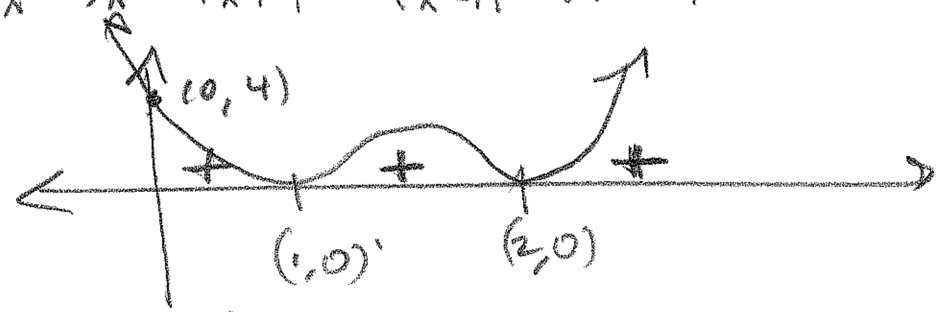
E.B. $6x^3$

signs alternate because all multiplicities are 1 and 1 is odd.

(69) $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4 = (x-1)^2(x+2)^2$

$f(0) = 4 \rightsquigarrow (0,4)$

x^4:



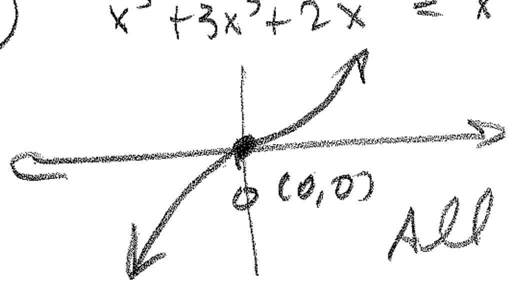
Never crosses x-axis,

$x=1, m=2$ Touch (not cross)

$x=2, m=2$ Touch

(70) $x^5 + 3x^3 + 2x = x(x-i)(x+i)(x-i\sqrt{2})(x+i\sqrt{2})$

EB: x^5



All we know is it crosses x-axis @ $x=0$ & its end behavior