121 §3.2 #s 9-15, 23, 29, 35-41, 47, 49
55, 57, 59, 61, 67, 77, 79, 81, 83

#s 5-10 Find Quotient & Remainder
ORDINARY DIVISION

(9)

$$\begin{array}{r} s^2 + 2 \;\; r16 \\ s^2-5 \,\overline{\big)\, s^4 + 0s^3 - 3s^2 + 6} \\ -\left(s^4 \qquad\quad - 5s^2\right) \\ \hline 2s^2 + 6 \\ (2s^2 - 10) \\ \hline 16 \end{array}$$

$s^2+2 =$ quotient
$16 =$ remainder

Two ways to interpret this:

$$\frac{s^4-3s^2+6}{s^2-2} = s^2+2+\frac{16}{s^2-2} \quad \text{or}$$

$$s^4-3s^2+6 = (s^2-2)(s^2+2)+16$$

(11) #s 11-22 SYNTHETIC DIVISION for quotient & remainder

$x^2+4x+1, \quad x-2$

$$x^2+4x+1 = (x+6)(x-2)+13$$

$$\begin{array}{r|rrr} 2 & 1 & 4 & 1 \\ & & 2 & 12 \\ \hline & 1 & 6 & 13 \end{array}$$

quotient: ~~x2~~ $x+6$
remainder: 13

121 §3.2 #s 13-15, 23, 29, 35-41, 47, 49, 55, 57, 58, 61, 67, 77, 79, 81, 83

(13) $-x^3+x^2-4x+9, \quad x+3$

$$\begin{array}{r|rrrr} -3 & -1 & 1 & -4 & 9 \\ & & -3 & 6 & -6 \\ \hline & 1 & -2 & 2 & 3 \end{array}$$

$\boxed{\begin{array}{l} x^2-2x+2 = \text{quotient} \\ 3 = \text{remainder} \end{array}}$

(15) $4x^3-5x+2, \quad x-\frac{1}{2}$

$$\begin{array}{r|rrrr} \frac{1}{2} & 4 & 0 & -5 & 2 \\ & & 2 & 1 & -2 \\ \hline & 4 & 2 & -4 & 0 \end{array}$$

$\boxed{\begin{array}{l} \text{quotient: } 4x^2+2x-4 \\ \text{remainder: } 0 \end{array}}$

(17) $2a^3-3a^2+4a+3, \quad a+\frac{1}{2}$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & -3 & 4 & 3 \\ & & -1 & 2 & -3 \\ \hline & 2 & -4 & 6 & 0 \end{array}$$

$\boxed{\begin{array}{l} \text{quotient: } 2x^2-4x+6 \\ \text{remainder: } 0 \end{array}}$

121 §3.2 #s 23, 29, 35-41, 47, 49, 55, 57, 58
61, 67, 77, 79, 81, 83

#s 23-34 $f(x)=x^5-1$, $g(x)=x^3-4x^2+8$, $h(x)=2x^4+x^3-x^2+3x+3$
Find function vals by synthetic Division

(23) $f(1)$

$$\begin{array}{r|rrrrrr} 1 & 1 & 0 & 0 & 0 & 0 & -1 \\ & & 1 & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 1 & \boxed{0 = f(1)} \end{array}$$

(29) $g(-\frac{1}{2})$

$64-9=55$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 1 & -4 & 0 & 8 \\ & & -\frac{1}{2} & \frac{9}{4} & -\frac{9}{8} \\ \hline & 1 & -\frac{9}{2} & \frac{9}{4} & \boxed{\frac{55}{8} = g(-\frac{1}{2})} \end{array}$$

#s 35-38 Determine if ~~it is~~ the binomial is a factor. If it is, then factor completely

(35) $x+3$, x^3+4x^2+x-6

$$\begin{array}{r|rrrr} -3 & 1 & 4 & 1 & -6 \\ & & -3 & -3 & 6 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$\boxed{\text{Yes!}}$

$x^2+x-2=0$
$(x+2)(x-1)=0$
$x=1,-2$

$\Rightarrow$ $\boxed{(x+3)(x-1)(x+2)}$

121 §3.2 #s 37–41, 47, 49, 55, 57, 58, 61, 67, 77, 79, 81, 83

37 $x-4, \; x^3+4x^2-17x-60$

$$\begin{array}{r|rrrr} 4 & 1 & 4 & -17 & -60 \\ & & 4 & 32 & 60 \\ \hline & 1 & 8 & 15 & 0 \end{array}$$ Yes!

$x^2+8x+15=0$

$(x+3)(x+5)=0$ $\longrightarrow$ $\boxed{(x-4)(x+5)(x+3)}$

$x \in \{-5, -3\}$

#s 39–46 Determine if # given is a zero of the given polynomial.

39 $3, \; f(x) = 2x^3-5x^2-4x+3$

$$\begin{array}{r|rrrr} 3 & 2 & -5 & -4 & 3 \\ & & 6 & 3 & -3 \\ \hline & 2 & 1 & -1 & 0 \end{array}$$ Yes

41 $-2, \; g(d) = d^3+2d^2+3d+1$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 3 & 1 \\ & & -2 & 0 & -6 \\ \hline & 1 & 0 & 3 & -5 \end{array}$$ No

121 §3.2 #s 47, 49, 55, 57, 58, 61, 67, 77, 79, 81, 83

#s 47–54 Find all possible rational zeros

(47) $f(x) = x^3 - 9x^2 + 26x - 24$

$p: 24 \qquad \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$q: 1$

(49) $h(x) = x^3 - x^2 - 7x + 15$

$p: 15 \qquad \pm 1, \pm 3, \pm 5, \pm 15$

$q: 1$

#s 55–78 Find all real and nonreal zeros

(55) $f(x) = x^3 - 9x^2 + 26x - 24$

$p: 24 \qquad \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$q: 1$

$$\begin{array}{r|rrrr} 1 & 1 & -9 & 26 & -24 \\ & & 1 & -8 & 18 \\ \hline & 1 & -8 & 18 & \text{No} \end{array} \qquad \begin{array}{r|rrrr} -1 & 1 & -9 & 26 & -24 \\ & & -1 & 10 & -36 \\ \hline & 1 & -10 & 36 & \text{No} \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -9 & 26 & -24 \\ & & 2 & -14 & 24 \\ \hline & 1 & -7 & 12 & 0 \end{array} \text{ Yes!}$$

$x^2 - 7x + 12 = 0$

$(x-3)(x-4) = 0$

$x = 3, 4$

$$\boxed{x \in \{2, 3, 4\}}$$

121 S' 3,2 #s 57, 58, 61, 67, 77, 79, 81, 83

57) $h(x) = x^3 - x^2 - 7x + 15$

$p: 15 \quad \pm 1, \pm 3, \pm 5, \pm 15$

$q: 1$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -7 & 15 \\ & & 1 & 0 & -7 \\ \hline & 1 & 0 & -7 & \text{NO} \end{array} \qquad \begin{array}{r|rrrr} -1 & 1 & -1 & -7 & 15 \\ & & -1 & 2 & 5 \\ \hline & 1 & -2 & -5 & \text{NO} \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -7 & 15 \\ & & 3 & 6 & -3 \\ \hline & 1 & 2 & -1 & \text{NO} \end{array} \qquad \begin{array}{r|rrrr} -3 & 1 & -1 & -7 & 15 \\ & & -3 & 12 & -15 \\ \hline & 1 & -4 & 5 & 0 \text{ Yes!} \end{array}$$

$x^2 - 4x + 5 = 0$

$a = 1,\ b = -4,\ c = 5$

$b^2 - 4ac = (-4)^2 - 4(1)(5)$

$= 16 - 20$

$= -4$

$\sqrt{-4} = 2i$

$x = \dfrac{4 \pm 2i}{2} = 2 \pm i$

$\boxed{x \in \{-3, 2 \pm i\}}$

 §3.2 #s 58, 61, 67, 77, 79, 81, 83

(58) $m(x) = x^3 + 4x^2 + 4x + 3$

$p: 3 \quad \pm 1, \pm 3$
$q: 1$

$$\begin{array}{r|rrrl} 1 & 1 & 4 & 4 & 3 \\ & & 1 & 5 & 9 \\ \hline & 1 & 5 & 9 & \text{NO} \end{array} \qquad \begin{array}{r|rrrl} -1 & 1 & 4 & 4 & 3 \\ & & -1 & -3 & -1 \\ \hline & 1 & 3 & 1 & \text{NO} \end{array}$$

$$\begin{array}{r|rrrl} 3 & 1 & 4 & 4 & 3 \\ & & 3 & 21 & 75 \\ \hline & 1 & 7 & 25 & \text{NO} \end{array} \qquad \begin{array}{r|rrrl} -3 & 1 & 4 & 4 & 3 \\ & & -3 & -3 & -3 \\ \hline & 1 & 1 & 1 & 0 \text{ Yes} \end{array}$$

$x^2 + x + 1 = 0$

$a = 1, b = 1, c = 1$

$b^2 - 4ac = 1^2 - 4(1)(1)$

$= 1 - 4$

$= -3$

$\sqrt{-3} = i\sqrt{3}$

$x = \dfrac{-1 \pm i\sqrt{3}}{2(1)} = \dfrac{-1 \pm i\sqrt{3}}{2}$

$\boxed{x \in \left\{-3, -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i\right\}}$

(61) $M(t) = 18t^3 - 21t^2 + 10t - 2$

$p: 2 \quad \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{9}, \pm \frac{1}{18}$
$q: 18 \quad \pm 2, \cancel{\pm \frac{2}{2}}, \pm \frac{2}{3}, \pm \frac{2}{9}, \cancel{\pm \frac{2}{18}}$

$$\begin{array}{r|rrrl} \frac{1}{2} & 18 & -21 & 10 & -2 \\ & & 9 & -6 & 2 \\ \hline & 18 & -12 & 4 & 0 \text{ Yes!} \end{array}$$

$18x^2 - 12x + 4 = 0$

$9x^2 - 6x + 2 = 0$

$a = 9, b = -6, c = 2$

$b^2 - 4ac = (-6)^2 - 4(9)(2)$

$= 36 - 72 = -36$

$\sqrt{-36} = 6i$

$x = \dfrac{6 \pm 6i}{2(18)} = \dfrac{6(1 \pm i)}{36}$

$= \dfrac{1 \pm i}{6}$

$\boxed{x \in \left\{\frac{1}{2}, \frac{1 \pm i}{6}\right\}}$

'21 §3.2 #s 67, 77, 79, 81, 83

(67) $V(x) = x^4 + 2x^3 - x^2 - 4x - 2$

$p: 2$ $\quad \pm 1, \pm 2$
$q: 1$

$$\begin{array}{r|rrrrr} -1 & 1 & 2 & -1 & -4 & -2 \\ & & -1 & -1 & 2 & 2 \\ \hline -1 & 1 & 1 & -2 & -2 & 0 \text{ Yes} \\ & & -1 & 0 & 2 & \\ \hline & 1 & 0 & -2 & 0 & \text{Yes} \end{array}$$

$x = -1, m = 2$

$x^2 - 2 = 0$
$x^2 = 2$
$x = \pm\sqrt{2}$

$x \in \{-1, \pm\sqrt{2}\}$

(77) $f(x) = (x^2 - 4x + 1)(x^3 - 9x^2 + 23x - 15)$
#1 #2

#1 $x^2 - 4x + 1 = 0$
$x^2 - 4x + 2^2 = -1 + 4$
$(x-2)^2 = 3$
$x - 2 = \pm\sqrt{3}$
$x = 2 \pm \sqrt{3}$

#2
$p: 15$ $\quad \pm 1, \pm 3, \pm 5, \pm 15$
$q: 1$

$$\begin{array}{r|rrrr} 3 & 1 & -9 & 23 & -15 \\ & & 3 & -18 & 15 \\ \hline & 1 & -6 & 5 & 0 \end{array}$$

$x = 3$

$x^2 - 6x + 5 = 0$
$(x-5)(x-1) = 0$
$x = 1, 5$

$$\boxed{x \in \{1, 3, 5, 2 \pm \sqrt{3}\}}$$

121 § 3.2 #s 79, 81, 83

#s 79-86 use division to write the rational expression in the form: quotient + $\frac{\text{remainder}}{\text{divisor}}$

79 $\frac{2x+1}{x-2} = f(x)$

$$\begin{array}{r|rr} 2 & 2 & 1 \\ & & 4 \\ \hline & 2 & 5 \\ & c & r \end{array} \longrightarrow \boxed{f(x) = 2 + \frac{5}{x-2}}$$

81 $\frac{a^2 - 3a + 5}{a-3} = f(a)$

$$\begin{array}{r|rrr} 3 & 1 & -3 & 5 \\ & & 3 & 0 \\ \hline & 1 & 0 & 5 \\ & a^1 & c & r \end{array} \longrightarrow f(a) = \boxed{a + \frac{5}{a-3}}$$

83 $f(c) = \frac{c^2 - 3c - 4}{c^2 - 4}$

$$\begin{array}{r|l} & 1 \quad r - 3c \\ c^2 - 4 & c^2 - 3c - 4 \\ & -(c^2 \quad\ \ - 4) \\ \hline & \quad -3c \end{array} \longrightarrow \boxed{f(c) = 1 - \frac{3c}{c^2 - 4}}$$