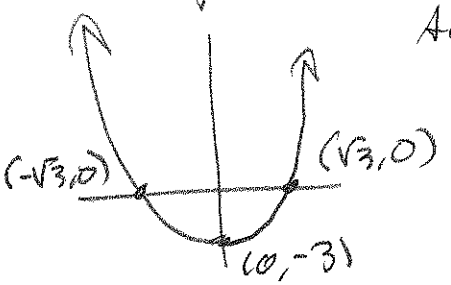
12.1 §3.1 #5 41-51 odds, 53-58 ALL, 65, 67, 71, 73

#'s 41-52 Axis of Symmetry, y-int, x-ints, sketch

(41) $y = x^2 - 3$

AOS: $x = 0$



$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

$a=1, b=0, c=-3$

$b^2 - 4ac =$

$0^2 - 4(1)(-3) = 12$

$\sqrt{12} = \sqrt{2 \cdot 2 \cdot 3}$

$= 2\sqrt{3}$

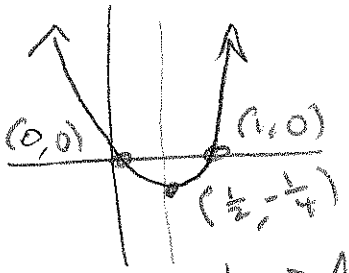
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{0 \pm 2\sqrt{3}}{2(1)} = \frac{\pm 2\sqrt{3}}{2} = \pm\sqrt{3}$

(43) $y = x^2 - x$

$= x(x-1) \overset{\text{SET}}{=} 0 \Rightarrow x = 0, 1$

M1



$x = \frac{1}{2}$ is AOS

TRICK: Vertex is always halfway between x-ints

$\frac{0+1}{2} = \frac{1}{2} = h$

$\left(\frac{1}{2}\right)^2 = \frac{1}{2} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} = k$

M2

$x^2 - x = x^2 - x + \left(\frac{1}{2}\right)^2 - \frac{1}{4}$

$= \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} \Rightarrow (h, k) = \left(\frac{1}{2}, -\frac{1}{4}\right)$

M3

$\overset{\text{SET}}{=} 0 \Rightarrow \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} = 0$

$\left(x - \frac{1}{2}\right)^2 = \frac{1}{4}$

$x - \frac{1}{2} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

$x = \frac{1}{2} \pm \frac{1}{2}$ → 1, 0

$a=1, b=-1, c=0$

$b^2 - 4ac$

$= (-1)^2 - 4(1)(0)$

$= 1$

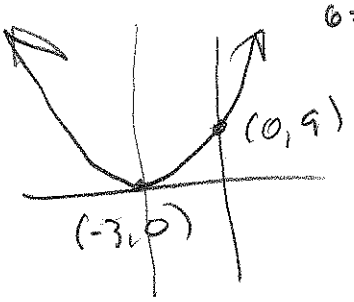
$\sqrt{1} = 1$

$x = \frac{1 \pm 1}{2}$ → $\frac{2}{2} = 1$, $\frac{0}{2} = 0$

12! §3.1 #s 45-51 odds, 53-58 ALL, 65, 67, 71, 73

(45) $f(x) = x^2+6x+9 = (x+3)^2$

$b=2\cdot 3 \quad 3^2=9$



$x=-3$ = AOS

(47) $f(x) = (x-3)^2-4 \quad (h,k) = (3,-4)$

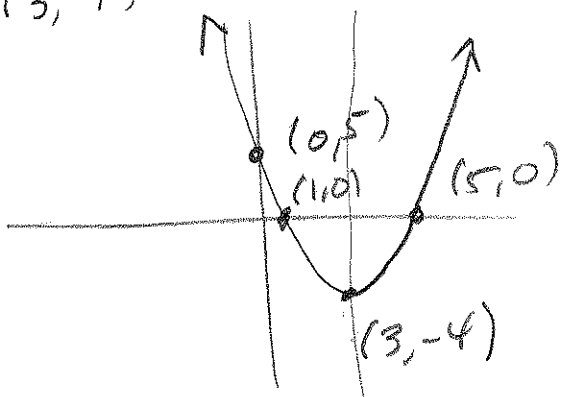
$(x-3)^2-4=0$

$(x-3)^2=4$

$x-3 = \pm\sqrt{4} = \pm 2$

$x = 3\pm 2$ → 5, 1

$f(0) = (-3)^2-4 = 9-4 = 5$



$x=3$ = AOS

M2

$(x-3)^2-4=0$

$x^2-6x+9-4=0$

$x^2-6x+5=0$

$(x-5)(x-1)=0$

$x\in\{1,5\}$

M3

$a=1, \quad b=-6, \quad c=5$

$b^2-4ac = (-6)^2-4(1)(5)$

$= 36-20$

$= 16$

$\sqrt{16} = 4$

$x = \frac{6\pm 4}{2} = 3\pm 2$ → 5, 1

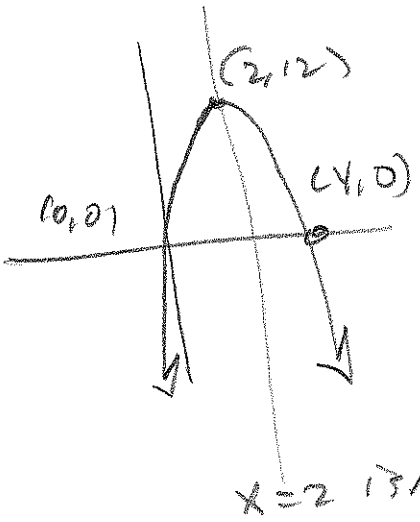
121 §3.1 #s41, 51, 53-58ALL, 65, 67, 71, 73

(49) $y = -3(x-2)^2 + 12$ $(h,k) = (2,12)$

$y(0) = -3(-2)^2 + 12$

$= -3(4) + 12$

$= 0$



$x = 2$ is AOS

M1 Since vertex is halfway between x-ints,

$x = 0$ is x-int

$x = 2$ is h

$x = 4$ is x-int

M2 $-3(x-2)^2 + 12 = 0$

$-3(x-2)^2 = -12$

$(x-2)^2 = \frac{-12}{-3} = 4$

$x - 2 = \pm 2$

$x = 2 \pm 2$ → 4, 0

M3 $-3(x^2 - 4x + 4) + 12$

$= -3x^2 + 12x - 12 + 12$

$= -3x^2 + 12x \stackrel{SET}{=} 0$

$-3x(x+4) = 0$

$x = 0, 4$

M4 $-3x^2 + 12x = 0$

$a = -3, b = 12, c = 0$

$b^2 - 4ac = 12^2 - 4(-3)(0)$

$= 144$

$\sqrt{144} = 12$

$x = \frac{-12 \pm 12}{2(-3)} = \frac{-12 \pm 12}{-6}$

$= \frac{12 \mp 12}{6} = 2 \mp 2$ → 0, 4

121 § 3.1 #s 51, 53-58ALL, 65, 67, 71, 73

(51) $y = -2x^2 + 4x + 1$

M1 $= -2(x^2 - 2x \quad) + 1$

$= -2(x^2 - 2x + 1^2) + 1 + 2(1)^2$

$= -2(x-1)^2 + 3$ $\quad (h,k) = (1,3)$

SET 0 $\Rightarrow$ $-2(x-1)^2 = -3$

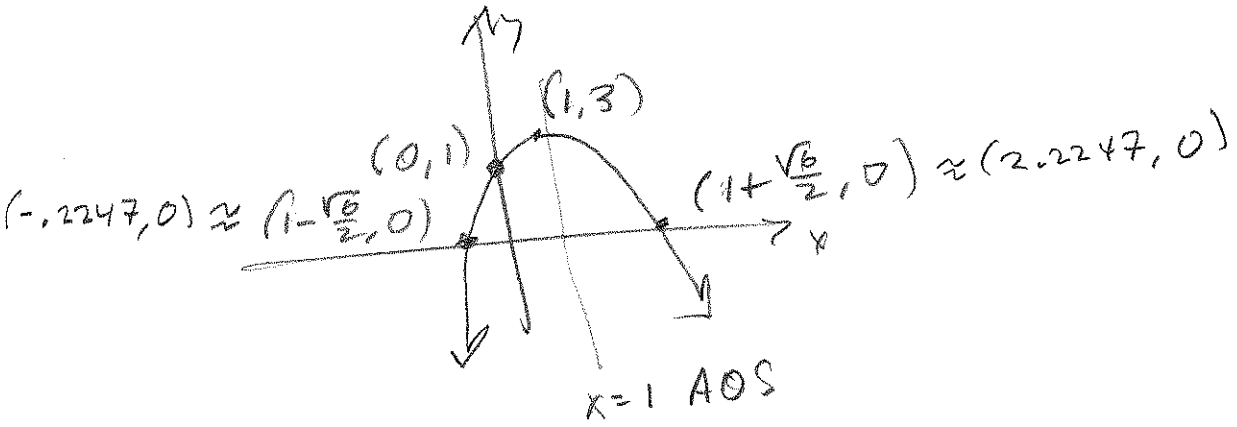
$(x-1)^2 = \frac{3}{2}$

$x - 1 = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{3}}{\sqrt{2}} = \pm\frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{6}}{2}$

$y(0) = 1$

$(0,1)$

$x = 1 \pm \frac{\sqrt{6}}{2}$



121 §3.1 #s 53-58 ALL, 65, 67, 71, 73

#s 53-59 Solve the inequality by graphing

(53) $x^2 - 2x - 3 > 0$

$(x-3)(x+1) > 0$

$x = -1, 3$

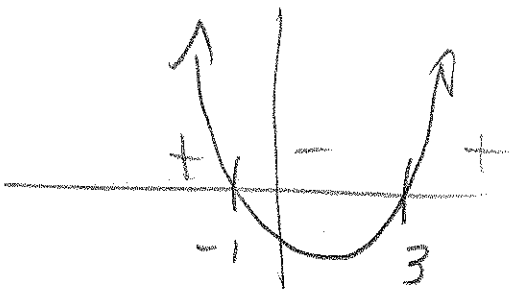
$a=1, b=-2, c=-3$

$b^2 - 4ac = (-2)^2 - 4(1)(-3)$

$= 4 + 12 = 16$

$\sqrt{16} = 4$

$x = \frac{2 \pm 4}{2(1)} = 1 \pm 2$ → 3, -1



want > 0

$(-\infty, -1) \cup (3, \infty)$

(54) $x^2 + x - 2 \geq 0$

$(x+2)(x-1) \geq 0$

$x \in \{-2, 1\}$

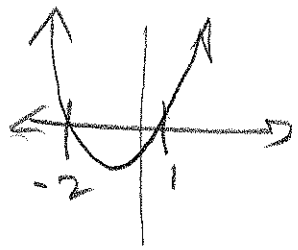
$a=1, b=1, c=-2$

$b^2 - 4ac = 1^2 - 4(1)(-2)$

$= 1 + 8 = 9$

$\sqrt{9} = 3$

$x = \frac{-1 \pm 3}{2(1)} = \frac{-1 \pm 3}{2}$ → $\frac{-4}{2} = -2$, $\frac{2}{2} = +1$



want ≥ 0

$(-\infty, -2] \cup [1, \infty)$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

121 §3.1 #55-58 ALL, 65, 67, 71, 73

(55) $x^2 - 4x + 1 < 0$

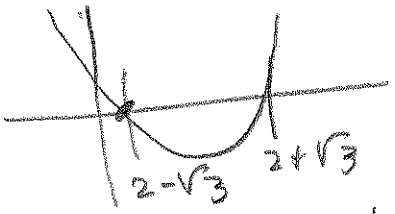
$a = 1, b = -4, c = 1$

$b^2 - 4ac = (-4)^2 - 4(1)(1)$

$= 16 - 4 = 12$

$\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$

$x = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$



Want < 0

$x \in (2-\sqrt{3}, 2+\sqrt{3})$

(56) $x^2 - 2x - 1 \leq 0$

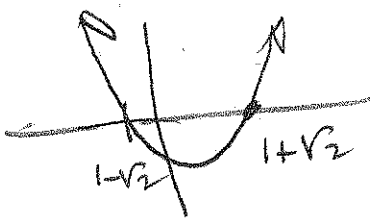
$a = 1, b = -2, c = -1$

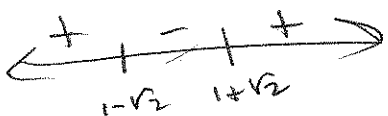
$b^2 - 4ac = (-2)^2 - 4(1)(-1)$

$= 4 + 4 = 8$

$\sqrt{8} = 2\sqrt{2}$

$x = \frac{2 \pm 2\sqrt{2}}{2(1)} = \frac{2(1 \pm \sqrt{2})}{2} = 1 \pm \sqrt{2}$





Want ≤ 0

$x \in [1-\sqrt{2}, 1+\sqrt{2}]$

121 §3.1 #s 57, 58, 65, 67, 71, 73

(57) $x+1 < 6x^2$

$-6x^2 + x - 1 < 0$

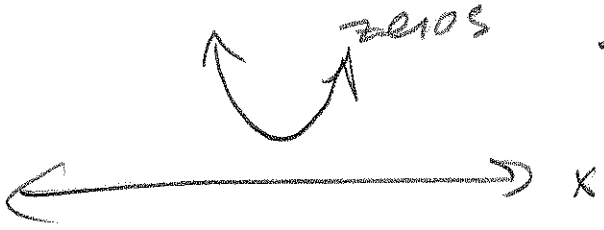
$6x^2 - x + 1 > 0$ New, equivalent problem

$a=6, b=-1, c=1$

$b^2-4ac = (-1)^2 - 4(6)(1)$

$= 1 - 24$

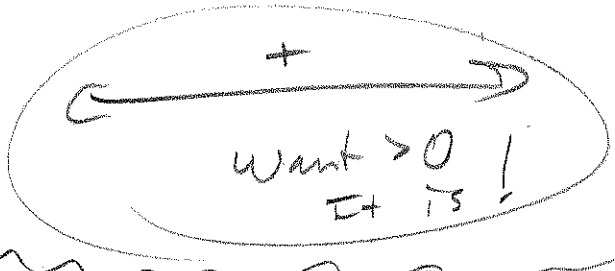
$= -23$ No real zeros



Say no more!

Always positive, since $+$ exponent

$\rightarrow 6x^2 - x + 1$



$x \in (-\infty, \infty)$

(58) $x+6 > 5x^2$

$-5x^2 + x + 6 > 0$

$5x^2 - x - 6 < 0$ New problem.

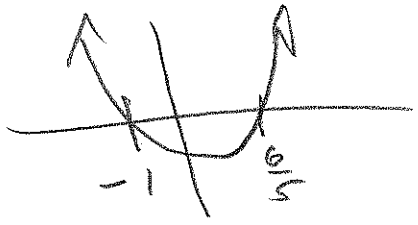
$a=5, b=-1, c=-6$

$b^2-4ac = (-1)^2 - 4(5)(-6)$

$= 1 + 120 = 121$

$\sqrt{121} = 11$ Damn thing factors!

$x = \frac{1 \pm 11}{2(5)} = \frac{1 \pm 11}{10}$ $\rightarrow \frac{12}{10} = \frac{6}{5}$, $\rightarrow \frac{-10}{10} = -1$



$x \in (-1, \frac{6}{5})$

121 §3.1 #s 65, 67, 71, 73

#s 65-76 Solve w/ Test pt method

(65) $x^2 - 4x + 2 < 0$

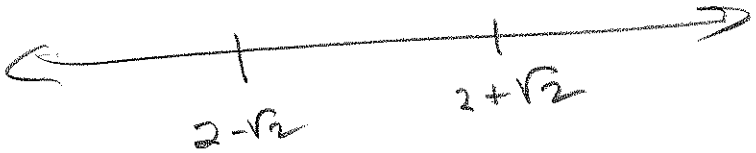
$a=1, b=-4, c=2$

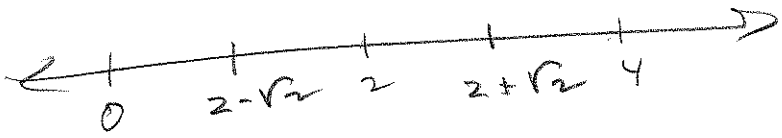
$b^2 - 4ac = (-4)^2 - 4(1)(2)$

$= 16 - 8 = 8$

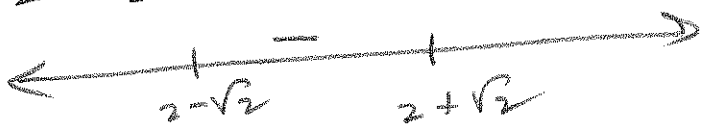
$\sqrt{8} = 2\sqrt{2} \approx 2.828$

$x = \frac{4 \pm 2\sqrt{2}}{2(1)} = \frac{2(2 \pm \sqrt{2})}{2} = 2 \pm \sqrt{2}$ → 3.4142, .5858

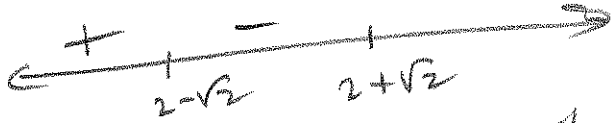




Test: $x=2$ $2^2 - 4(2) + 2 = 4 - 8 + 2 = -2$ negative



$x=0$; $0^2 - 4(0) + 2 = 2$ Positive:



$x=4$; $4^2 - 4(4) + 2 = 2$ Positive

Want < 0

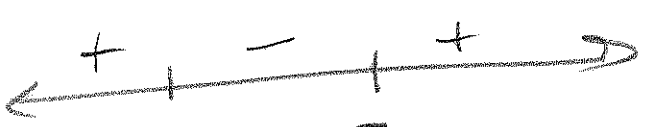
$(2-\sqrt{2}, 2+\sqrt{2})$ FINAL ANS.

121 §3.1 #s 67, 71, 73

(67) $x^2 - 9 > 1$

$x^2 - 10 > 0$

$x = \pm\sqrt{10}$ is critical



want > 0

$(-\infty, -\sqrt{10}) \cup (\sqrt{10}, \infty)$

Test: -4 $(-4)^2 - 9 = 16 - 9 > 0$ +

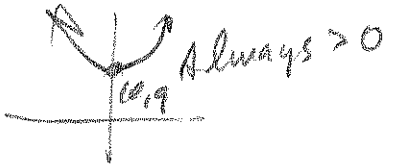
0 $0^2 - 9 = -9$ −

$+4$ $4^2 - 9 = 16 - 9 = 7$ +

(71) $p^2 + 9 > 0$

$1^2 + 9 = 10 > 0$

so always



$(-\infty, \infty)$

(73) $a^2 + 20 \leq 8a$

$a^2 - 8a + 20 \leq 0$

Always + or Always −

$a = 1, b = -8, c = 20$

want ≤ 0

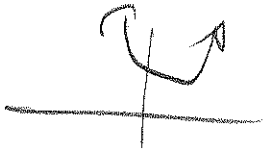
$b^2 - 4ac = (-8)^2 - 4(1)(20)$

$= 64 - 80 = -16$ No zeros

$f(0) = 20 > 0$

so Always + (> 0)

Never − (< 0)



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