121 §2.5 #s 1-7, 11-27, 41, 51-57, 67-81

(1) If a function has no two ordered pairs with the same 2nd coordinates, then the function is <u>1-to-1</u>

(2) A 1-to-1 function is <u>invertible</u>.

(3) If f's invertible, then the function obtained by swapping x's & y's is the <u>inverse</u> of f.

(4) The graphs of f & f^{-1} are symmetric about the line <u>$y=x$</u>.

#s 5-10 Determine if each is 1-to-1:

(5) $\{(3,3), (5,5), (6,6), (9,9)\}$ $\boxed{\text{Yes}}$

(6) $\{(3,4), (5,6), (7,8), (9,10), (11,15)\}$ $\boxed{\text{Yes}}$

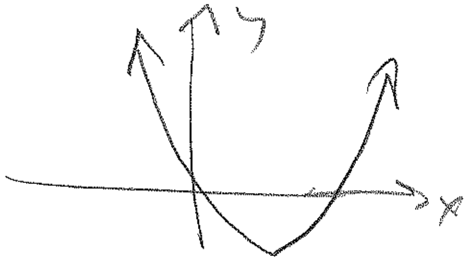
(7) $\{(-1,1), (1,1), (-2,4), (2,4)\}$ Not 1-to-1

It's a function, but $(-2,4), (2,4)$ make it not 1-to-1. So do $(-1,1)$ & $(1,1)$

#s 11-16 Use horizontal line test to determine if the function is 1-to-1.

121 § 2.5 #s 11-27, 41, 51-57, 67-81

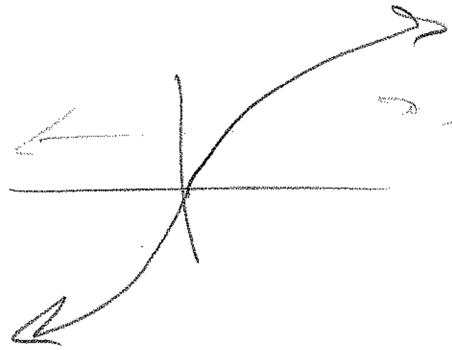
(11)



x-axis shows failed HLT. $\boxed{\text{NOT}}$ 1-to-1.

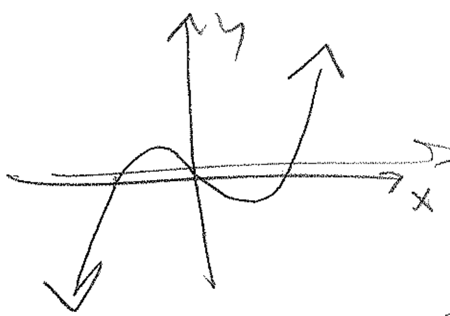
$f(x)=x^2-3x$

(13) $y=\sqrt[3]{x}+2$



$\boxed{\text{Yes.}}$ 1-to-1.

(15) $y = x^3 - x = x$



Not 1-to-1

#s 17-26 Determine if f is 1-to-1.

(17) $f(x)=2x-3$

$2x_1-3=2x_2-3$

$2x_1=2x_2$

$x_1=x_2$ Yes 1-to-1.

121 §2.5 #s 19-27, 41, 51-57, 67-81

(19) $g(x) = \frac{1-x}{x-5}$

$$\frac{1-x_1}{x_1-5} = \frac{1-x_2}{x_2-5}$$

$$(1-x_1)(x_2-5) = (1-x_2)(x_1-5)$$

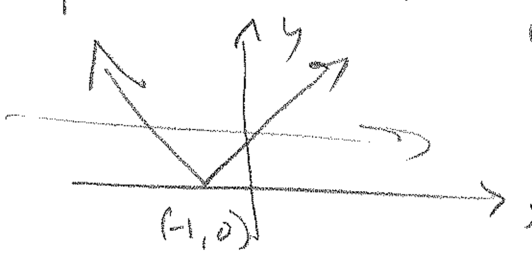
$$x_2 \underline{-5} \underline{-x_1x_2} + 5x_1 = x_1 \underline{-5} \underline{-x_2x_1} + 5x_2$$

$$x_2 + 5x_1 = x_1 + 5x_2$$

$$4x_1 = 4x_2$$

$$x_1 = x_2 \quad \boxed{\text{Yes}} \text{ 1-to-1}$$

(21) $p(x) = |x+1|$



$\boxed{\text{Not}}$ 1-to-1 by graph

$$|x_1+1| = |x_2+1|$$

$x_1+1 = x_2+1$ OR $x_1+1 = -(x_2+1)$

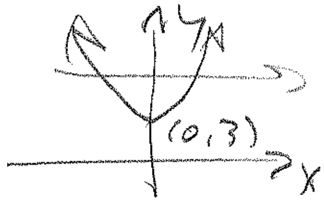
$\underline{x_1 = x_2}$ OR $\underline{x_1 = -x_2}$

⇓ Two places where $f(x_1) = f(x_2)$

∴ <u>NOT</u> one-to-one

121 §2.5 #s 23-27, 41, 51-57, 67-81

(23) $w(x) = x^2 + 3$ $\boxed{\text{Not}}$ 1-to-1 by graph



$$x_1^2 + 3 = x_2^2 + 3$$

$$x_1^2 = x_2^2$$

$$\sqrt{x_1^2} = \sqrt{x_2^2}$$

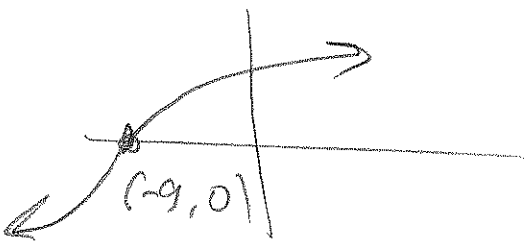
$$|x_1| = |x_2|$$

$$x_1 = \pm x_2$$

see? 2 places?

(25) $h(x) = \sqrt[3]{x+9}$

$\boxed{\text{Yes}}$ 1-to-1 by graph



$$\sqrt[3]{x_1 + 9} = \sqrt[3]{x_2 + 9}$$

$$x_1 + 9 = x_2 + 9$$

$$x_1 = x_2 \checkmark$$

#s 27-34 Determine if the function is invertible. If it is, then invert it.

(27) $\{(9,3), (2,2)\}$ Yes, $f^{-1} = \{(3,9), (2,2)\}$

#s 41-44 Find f^{-1}, $f^{-1}(5)$ & $(f^{-1} \circ f)(2)$

(41) $f = \{(2,1), (3,5)\} \Longrightarrow f^{-1} = \{(1,2), (5,3)\}$

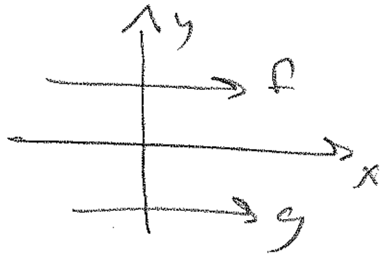
$f^{-1}(5) = 3$

$(f^{-1} \circ f)(2) = f^{-1}(f(2)) = f^{-1}(1) = 2$

121 §2.5 #s ~~43-47~~, 51-57, 67-81

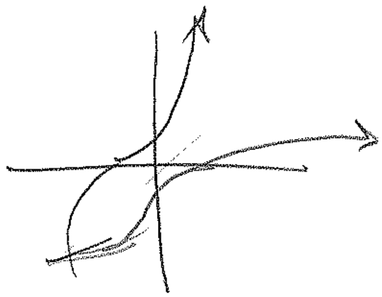
~~51~~ #s 51-54 Determine if they inverses:

51



No

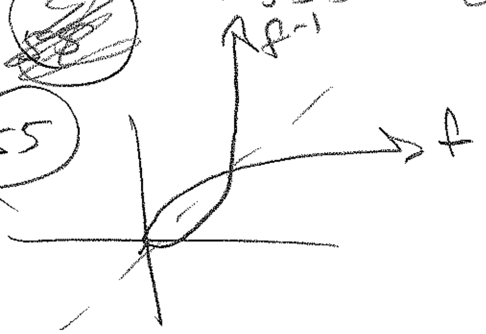
53



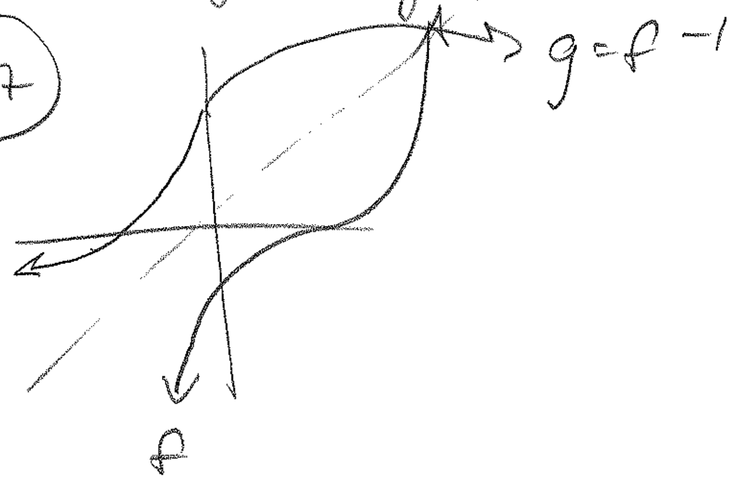
Yes

#s 55-58 Sketch f^{-1} given graph of f.

55



57



#s 67-81. Find $f^{-1}(x)$ by switch-and-solve.

67 $f(x) = 3x - 7$

$\cancel{x=}\ 3y - 7 = x$

$3y = x + 7$

$\boxed{y = \frac{x+7}{3}}$

69 $f(x) = 2 + \sqrt{x-3}$ for $x \geq 3$, $y \geq 2$

$2 + \sqrt{y-3} = x$

$\sqrt{y-3} = x - 2$

$y - 3 = (x-2)^2$

$\boxed{y = (x-2)^2 + 3}$
Keep $x \geq 2$

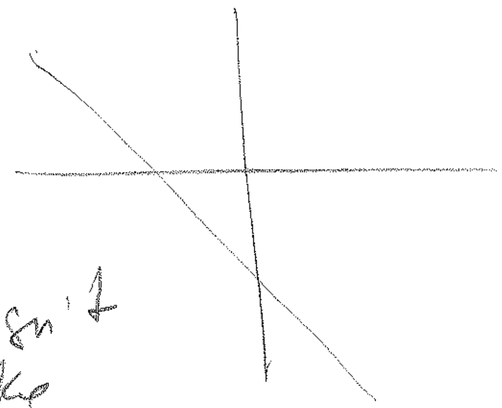
121 §2.5 #s 71-81

71 $f(x) = -x-9$

$-y-9 = x$

$-y = x+9$

$y = -x-9$ Doesn't make sense oh I see. Yep.



$f^{-1}(x) = -x-9$

$f(f^{-1}(3))$

$= f(-3-9)$

$= f(-12)$

$= -(-12)-9$

$= 12-9 = 3$ ✓

73 $f(x) = \dfrac{x+3}{x-5}$

$\dfrac{y+3}{y-5} = x$

$y-3 = xy-5x$

$y-xy = -5x+3$

$y(1-x) = -5x+3$

$y = \dfrac{-5x+3}{1-x}$ OR $\dfrac{5x-3}{x-1} = f^{-1}(x)$

75 $f(x) = -\dfrac{1}{x}$

$-\dfrac{1}{y} = x$

$-1 = xy$

$\boxed{-\dfrac{1}{x} = y = f^{-1}(x)}$

121 §2.5 #s 77-81

77 $f(x) = \sqrt[3]{x-9} + 5$

$\sqrt[3]{y-9} + 5 = x$

$\sqrt[3]{y-9} = x-5$

$y-9 = (x-5)^3$

$y = \boxed{(x-5)^3 + 9 = f^{-1}(x)}$

79 $f(x) = (x-2)^2$ for $x \geq 2$ & $y \geq 0$

So inverse will be $x \geq 0$ & $y \geq 2$

$(y-2)^2 = x$

$y-2 = \pm\sqrt{x}$

$y = \pm\sqrt{x} + 2$

So keep it ≥ 2 by picking $\boxed{f^{-1}(x) = \sqrt{x} + 2}$

81 Find $f(g(x))$ & $g(f(x))$. Are they inverses?

$f(x) = 4x+4$, $g(x) = .25x - 1$

$f(g(x)) = 4g + 4 = 4(.25x-1) + 4 = x - 4 + 4 = x$

Yes!

$g(f(x)) = .25f - 1 = .25(4x+4) - 1$

$= x + 1 - 1$

$= x$ ✓