121 CHAPTER 1 TEST — 1st Assignment

#s 1-8 Find all real or imaginary sol'ns.

(1) $2x+1 = x-6$

$\underline{-x-1 \quad -x+1}$

$x = -7 \longrightarrow$

$\boxed{x \in \{-7\}}$

(2) $\frac{1}{2}x - \frac{1}{6} = \frac{1}{3}$ LCD $= 2 \cdot 3 = 6$

$\frac{x}{2} \cdot \frac{3}{3} - \frac{1}{6} = \frac{1}{3} \cdot \frac{2}{2}$

$\frac{3x-1}{6} = \frac{2}{6}$

$3x - 1 = 2$

$3x = 3$

$x = 1 \longrightarrow \boxed{x \in \{1\}}$

(3) $3x^2 - 2 = 0$

(M1) $3x^2 = 2$

$x^2 = \frac{2}{3}$

$x = \pm\sqrt{\frac{2}{3}}$

$= \pm\sqrt{\frac{2}{3} \cdot \frac{3}{3}}$

$= \pm \frac{\sqrt{6}}{\sqrt{3^2}} = \pm \frac{\sqrt{6}}{3}$

$\longrightarrow \boxed{x \in \{\pm \frac{\sqrt{6}}{3}\}}$

(M2) $a=3, b=0, c=-2$

$b^2 - 4ac = 0 - 4(3)(-2) = 24$

$\sqrt{24} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3} = 2\sqrt{6}$

$2 \lfloor 24$
$2 \lfloor 12$
$2 \lfloor 6$
3

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{0 \pm 2\sqrt{6}}{2(3)} = \pm \frac{2\sqrt{6}}{2(3)} = \pm \frac{\sqrt{6}}{3}$

$\longrightarrow \boxed{x \in \{\pm \frac{\sqrt{6}}{3}\}}$

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(3) (M3) $3x^2-2=0 \Longrightarrow \sqrt{3}^2x^2-\sqrt{2}^2=0$

Sometimes it can be helpful to "see" difference of two "squares" more broadly.

$$\Longrightarrow (\sqrt{3}\,x)^2-\sqrt{2}^2=0$$

$$\Longrightarrow (\sqrt{3}\,x-\sqrt{2})(\sqrt{3}\,x+\sqrt{2})=0$$

$$\Longrightarrow \sqrt{3}\,x-\sqrt{2}=0 \quad \text{OR} \quad \sqrt{3}\,x+\sqrt{2}=0$$

$$\Longrightarrow \sqrt{3}\,x=\sqrt{2} \quad \text{OR} \quad \sqrt{3}\,x=-\sqrt{2}$$

$$\Longrightarrow x=\frac{\sqrt{2}}{\sqrt{3}} \quad \text{OR} \quad x=-\frac{\sqrt{2}}{\sqrt{3}}$$

$$\Longrightarrow x=\frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} \quad \text{OR} \quad x=-\frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

$$\Longrightarrow x=\frac{\sqrt{6}}{3} \quad \text{OR} \quad x=-\frac{\sqrt{6}}{3}$$

$$\Longrightarrow x\in\left\{\pm\frac{\sqrt{6}}{3}\right\}$$

(4) (M1)

$x^2+1=6x$

$x^2-6x=-1$

$2\,|\,8$, $2\,|\,4$, 2

$x^2-6x+3^2=-1+9$

$(x-3)^2=8$

$x-3=\pm\sqrt{8}=\pm\sqrt{2\cdot2\cdot2}=\pm2\sqrt{2}$

$x=3\pm2\sqrt{2}$

$x\in\{3\pm2\sqrt{2}\}$

(M2) $x^2-6x+1=0$

$a=1,\ b=-6,\ c=1$

$b^2-4ac=(-6)^2-4(1)(1)$

$=36-4=32$

$2\,|\,32$, $2\,|\,16$, $2\,|\,8$, $2\,|\,4$, 2

$\sqrt{32}=\sqrt{2^5}=\sqrt{2^4\cdot2}$

$=2^2\sqrt{2}=4\sqrt{2}$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{6\pm4\sqrt{2}}{2(1)}$$

$$=\frac{2(3\pm2\sqrt{2})}{2}=3\pm2\sqrt{2}$$

$\Longrightarrow x\in\{3\pm2\sqrt{2}\}$

The $\sqrt{\ }$ in the answer says factoring by ac doesn't work.

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5 $x^2+14=9x$

M1

$x^2-9x+\ ? = -14$

$x^2-9x+\left(\frac{9}{2}\right)^2=-14+\frac{81}{4}$

$\left(x-\frac{9}{2}\right)^2=\frac{-56+81}{4}=\frac{25}{4}$

$x-\frac{9}{2}=\pm\sqrt{\frac{25}{4}}=\pm\frac{5}{2}$

$x=\frac{9}{2}\pm\frac{5}{2}$ $\frac{14}{2}=7$ $\frac{4}{2}=2$

$\boxed{x\in\{2,7\}}$

M2

$x^2-9x+14=0$

$b^2-4ac=(-9)^2-4(1)(14)$
$=81-56=25$

$\sqrt{25}=5$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{9\pm5}{2}$ 7 2

$x\in\{2,7\}$

Two ac Methods:

M3

$\begin{array}{ccc} a & b & c \\ 1x^2 & -9x & +14 \end{array}$

Factors of $(14)(1)=14=ac$
whose sum is -9

$(-2)(-7)=14,\ -2-7=-9$ ✓

$x^2-7x-2x+14=0$

$x(x-7)-2(x-7)=0$

$(x-7)(x-2)=0$

$x\in\{2,7\}$

Want to find #s
whose sum is -9
and whose product
is $ac=(14)(1)=14$

$-9=-8-1$ 8

$-9=-7-2$ 14 ✓

REPEAT

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6 $\dfrac{x-1}{x+3} = \dfrac{x+2}{x-6}$ LCD $= (x+3)(x-6)$

$$\left(\frac{x-1}{x+3}\right)\left(\frac{x-6}{x-6}\right) = \left(\frac{x+2}{x-6}\right)\left(\frac{x+3}{x+3}\right)$$

$$\frac{x^2-7x+6}{LCD} = \frac{x^2+5x+6}{LCD}$$

HERE'S WHERE THE METHOD BRANCHES FOR INEQUALITIES

$$x^2 - 7x + 6 = x^2 + 5x + 6$$

$$-7x + 6 = 5x + 6$$

$$\underline{-5x - 6 = -5x - 6}$$

$$-12x = 0$$

$$\frac{-12x}{-12} = \frac{0}{-12}$$

$$x = 0$$

$$\boxed{x \in \{0\}}$$

7 $x^2 = 2x - 5$

M1

$$x^2 - 2x = -5$$

$$x^2 - 2x + 1^2 = -5 + 1^2$$

$$(x-1)^2 = -4$$

$$x - 1 = \pm\sqrt{-4} = \pm i\sqrt{4} = \pm 2i$$

$$x = 1 \pm 2i$$

$$x \in \{1 \pm 2i\}$$

M2 $x^2 - 2x + 5 = 0$

$$b^2 - 4ac = (-2)^2 - 4(1)(5)$$

$$= 4 - 20 = -16$$

$$\sqrt{-16} = i\sqrt{16} = 4i$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{2 \pm 4i}{2(1)} = \frac{2(1 \pm 2i)}{2} = 1 \pm 2i$$

$\longrightarrow x \in \{1 \pm 2i\}$

Doesn't Factor by ac.

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8

M1

$$x^2+1=0$$
$$x^2=-1$$
$$x=\pm\sqrt{-1}=\pm i$$
$$\boxed{x\in\{\pm i\}}$$

M2

$a=1,\ b=0,\ c=1$

$$b^2-4ac=0^2-4(1)(1)=-4$$
$$\sqrt{-4}=2i$$
$$x=\frac{0\pm 2i}{2(1)}=\pm\frac{2i}{2}=\pm i$$
$$\boxed{x\in\{\pm i\}}$$

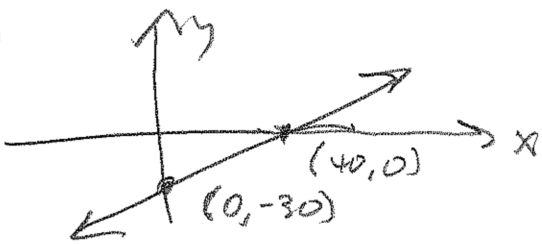
M3 Have to be twisted to "see" this as a difference of two squares:

$$x^2+1=x^2-(-1)=x^2-\sqrt{-1}^{\,2}=x^2-i^2$$
$$=(x-i)(x+i)\overset{\text{SET}}{=}0\implies\boxed{x\in\{\pm i\}}\ !$$

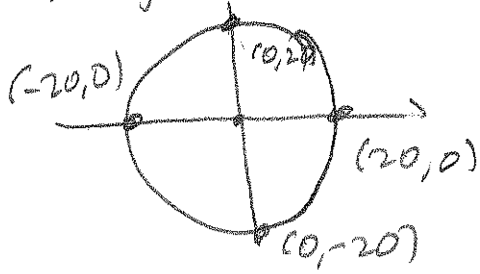
#s 9–14 SKETCH

9 $3x-4y=120$

x	y
0	-30
40	0



10 $x^2+y^2=400=20^2$

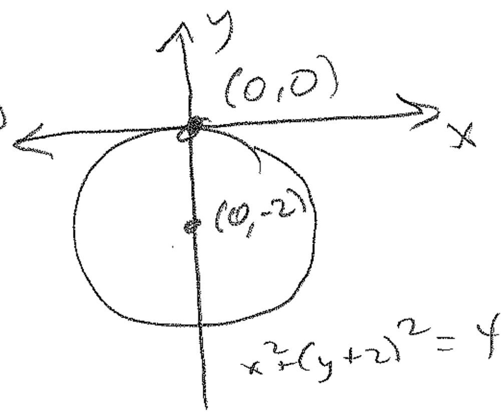


11 $x^2+y^2+4y=0$

$$x^2+y^2+4y+2^2=4$$
$$x^2+(y+2)^2=2^2$$

$(h,k)=(0,-2)$

$r=2$



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12 $y = -\frac{2}{3}x + 4$

$\Rightarrow (0,4) = y\text{-int}$

$y = -\frac{2}{3}x + 4 \overset{\text{SET}}{=} 0 = y$

LCD = 3

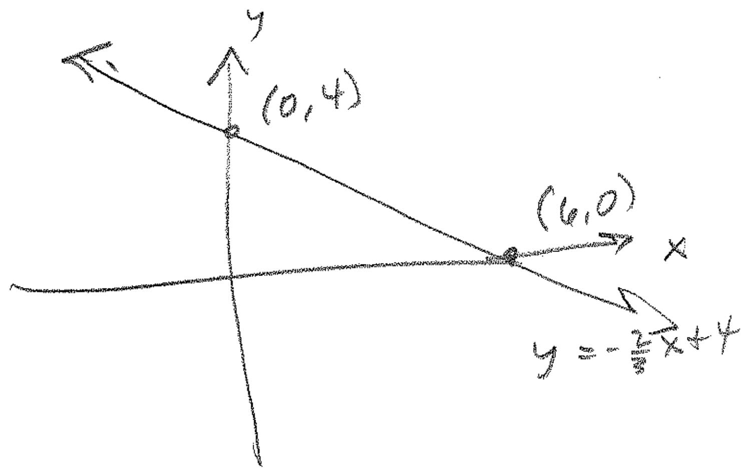
$-\frac{2}{3}x + 4 = 0$

$-\frac{2x}{3} + \frac{4}{1} \cdot \frac{3}{3} = \frac{0}{3}$

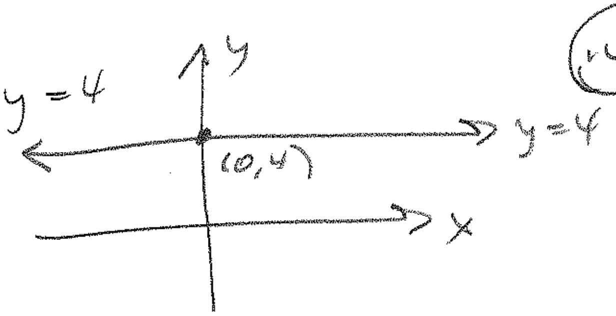
$-2x + 12 = 0$

$-2x = -12$

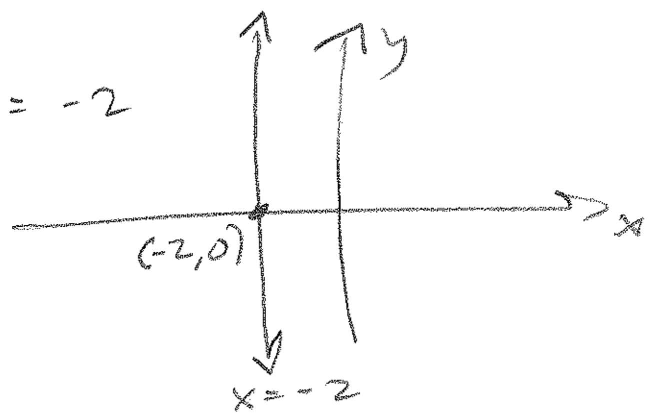
$x = 6 \rightsquigarrow (6,0)$ x-int



13 $y = 4$



14 $x = -2$



#s 15-22 Solve!

15 Slope of $3x - 5y = 8$ is $\boxed{\frac{3}{5} = m}$

To see this:

$-5y = -3x + 8$

$y = -\frac{3x}{-5} + \frac{8}{-5}$

$y = \frac{3}{5}x - \frac{8}{5}$

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(16) Slope thru $(-3,6)$ & $(5,-4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-6}{5-(-3)} = \frac{-10}{8} = \boxed{-\frac{5}{4} = m}$$

(17) Slope-int. form of line thru $(1,-2)$ & perpendicular to $2x - 3y = 6$ "$\perp$"

$m = +\frac{2}{3} \longrightarrow m_{\perp} = -\frac{3}{2}$ is what we want.

So $y = m(x - x_1) + y_1$ becomes

$$y = -\frac{3}{2}(x-1) - 2$$

$$y = -\frac{3}{2}x + \frac{3}{2} - \frac{4}{2}$$

$$\boxed{y = -\frac{3}{2}x - \frac{1}{2}}$$

(18) Eq'n of line thru $(3,-4)$ that's parallel to the line thru $(0,2)$ & $(3,-1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-2}{3-0} = \frac{-3}{3} = -1 = m$$

$$y = m(x - x_1) + y_1$$

$$y = -1(x-3) - 4$$

$$y = -x + 3 - 4$$

$$\boxed{y = -x - 1}$$

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(19) $P(-3,1)$ & $Q(2,4) \longrightarrow D(P,Q)$

$$= \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$
$$= \sqrt{(-3-2)^2 + (1-4)^2}$$
$$= \sqrt{(-5)^2 + (-3)^2}$$
$$= \sqrt{25+9} = \boxed{\sqrt{34} = D(P,Q)}$$

(20) Find midpoint M of segment between $P(-1,1)$ & $Q(1,0)$?

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-1+1}{2}, \frac{1+0}{2}\right) = \boxed{\left(0, \tfrac{1}{2}\right) = \text{Midpt}}$$

(21) $x^2-5x+9=0 \longrightarrow a=1, b=-5, c=9 \longrightarrow$

Discriminant $= b^2-4ac = (-5)^2-4(1)(9) = 25-36$
$= -9$

$b^2-4ac = -9 \Longrightarrow$ $\boxed{\text{No real solutions}}$

(22) Solve $5-2y = 4+3xy$ for y

$$-2y-3xy = -1$$
$$2y+3xy = 1$$
$$y(2+3x) = 1 \longrightarrow \boxed{y = \frac{1}{2+3x}}$$

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#s 23-26 Solve the inequalities. State sol'n set with interval notation & graph on # line.

(23) $3 - 2x > 7$

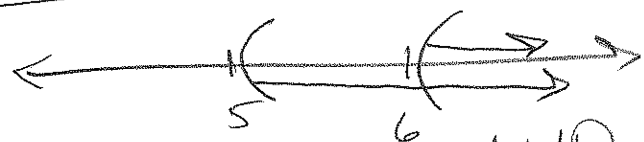
$-2x > 4$

$x < -2$

$x \in (-\infty, -2)$

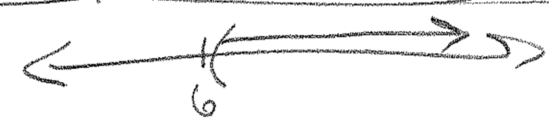
(24) $\frac{x}{2} > 3$ AND $5 < x$

$x > 6$ AND $x > 5$



AND

$x \in (6, \infty)$

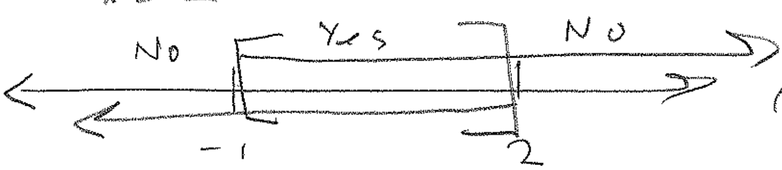


(25) $2x - 1 \le 3$

$2x-1 \le 3$ and $2x-1 \ge -3$

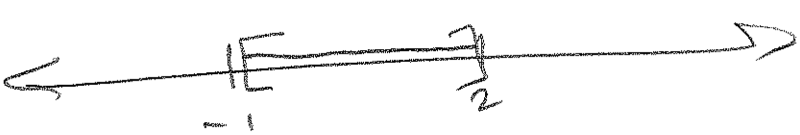
$2x \le 4$ and $2x \ge -2$

$x \le 2$ and $x \ge -1$

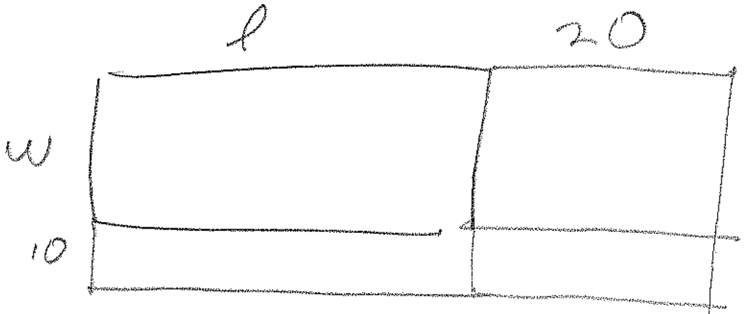


AND

$x \in [-1, 2]$



(27) Square patio. After expanding length by 20, width by 10, the area is 999 ft^2. What was its original area?



l = length (ft)
w = width (ft)

$(l+20)(w+10)=999$ Wait! It's square!

$l = w$!

$(w+20)(w+10)=999$

$w^2+30w+200=999$

$w^2+30w = 799$

$w^2+30w+15^2 = 799+225 = 1024 = 2^{10}$

$(\sqrt{2^{10}} = 2^{\frac{10}{2}} = 2^5$

$(w+15)^2 = 2^{10}$

$w+15 = \pm 2^5 = \pm 32$ $\boxed{17 = w}$

$w = -15 \pm 32 \to 17, -47$

$w = 17$, so original AREA was

$w^2 = 17^2 = \boxed{289 \text{ ft}^2}$

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(28) How many gallons of 20% alcohol must be mixed with 10 gal of 50% alcohol for 30% alcohol solution?

Amt Pure Alcohol = Amt Pure Alcohol

$.2x + .5(10) = .3(x+10)$, where

x = # of gallons of 20% alcohol.

$\Rightarrow .2x + 5 = .3x + 3$

$\Rightarrow -.1x = -2$

$\Rightarrow$ $\boxed{x = \frac{2}{.1} = 20 \text{ gal of 20\% alcohol}}$

(29) Price was \$311,000 in 1997 & \$495,000 in 2007. Model price as linear function of years after 1997.

Let x = # of years after 1997

y = Price (in thousands of dollars)

(a) We build the line thru $(x_1, y_1) = (0, 311)$ and $(x_2, y_2) = (10, 495)$ (2007-1997 = 10 years after 1997)

$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{495 - 311}{10 - 0} = \frac{184}{10} = 18.4 \Rightarrow$ $\boxed{y = 18.4x + 311}$

$y = 18.4(x - 0) + 311 \Rightarrow$

(b) Price in 2015 : 2015 - 1997 = 18 $\Rightarrow$

$y = 18.4(18) + 311 = \boxed{642.2 = y}$ $\boxed{\$642,200 \text{ in } 2015}$