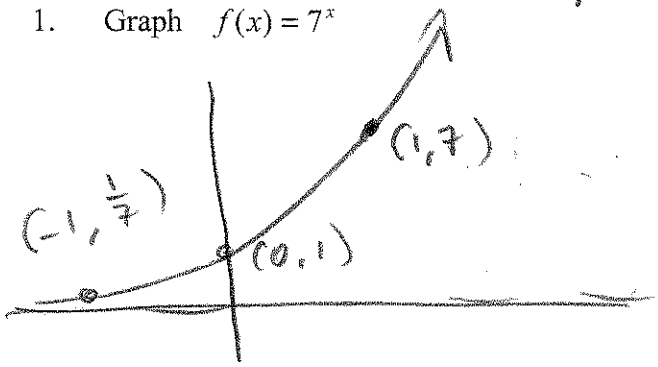
MAT 121 100 Points Fall, 2012

Test 4, Chapter 4 #8: Millenium #10: 7 years Name KEY

1. Graph $f(x) = 7^x$

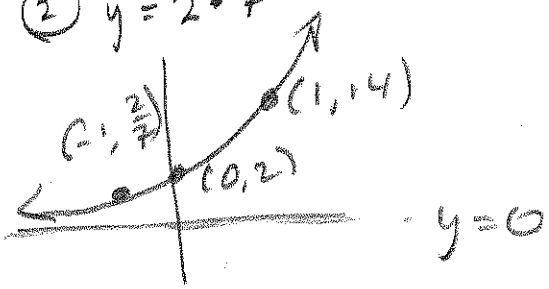


10pts

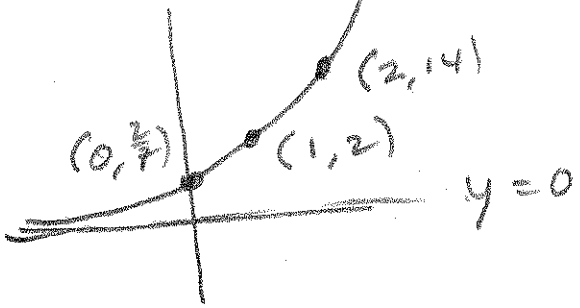
2. Graph $g(x) = 2 \cdot 7^{x-1} - 3$ by transforming the basic function $f(x) = 7^x$

① $y = 7^x$ See #1

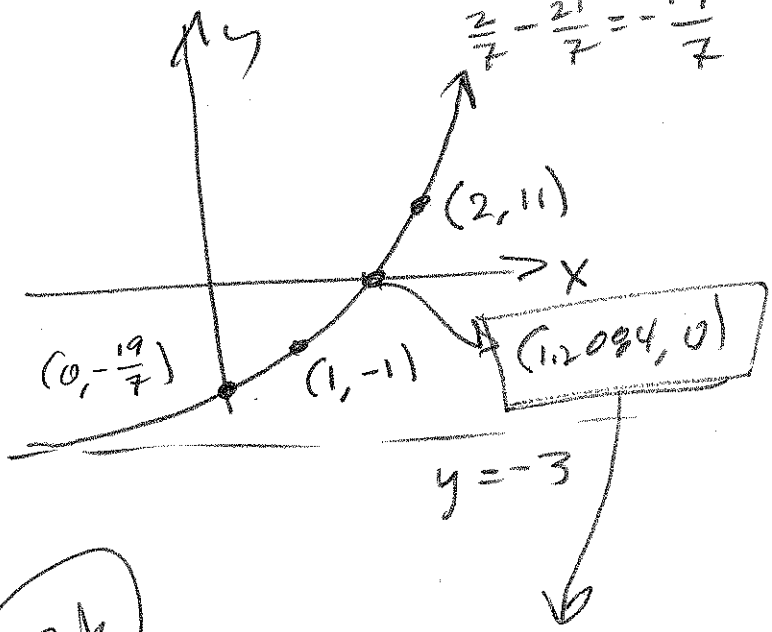
② $y = 2 \cdot 7^x$



③ $y = 2 \cdot 7^{x-1}$



④ $g(x) = 2 \cdot 7^{x-1} - 3$



5pts

Bonus

.5

Bonus Find the inverse of the function $g(x) = 2 \cdot 7^{x-1} - 3$

$2 \cdot 7^{y-1} - 3 = x$

$2 \cdot 7^{y-1} = x + 3$

$7^{y-1} = \frac{x+3}{2}$

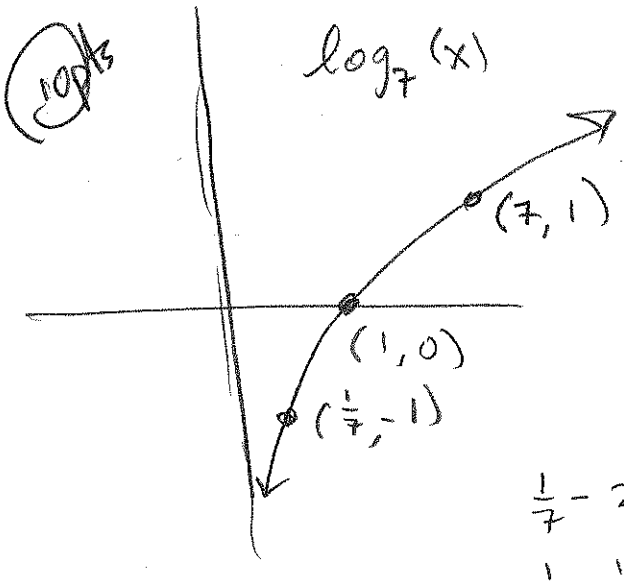
$y - 1 = \log_7\left(\frac{x+3}{2}\right)$

$y = \boxed{\log_7\left(\frac{x+3}{2}\right) + 1 = f^{-1}(x)}$

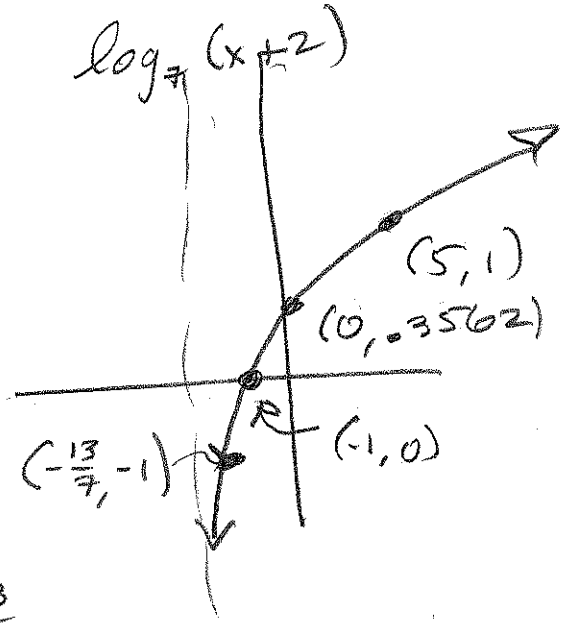
5pts

3. Graph $h(x) = \log_7(x+2)$

10pts



$\frac{1}{7} - 2 = \frac{1}{7} - \frac{14}{7} = -\frac{13}{7}$



$\log_7(0+2)$

$= \log_7(2)$

$= \frac{\ln 2}{\ln 7} \approx .3562071871$

.0

4. Solve $\log_5(x-4)+\log_5(x+2)=\log_5(7)$ for x.

10pts

$\log_5((x-4)(x+2)) = \log_5(7)$

$x^2-2x-8=7$

$x^2-2x-15=0$

$(x-5)(x+3)=0$

$x=5$ or $x=-3$

$\log_5(5-4)+\log_5(7)$

$=\log_5(1)+\log_5(7)$

$=\log_5(7)$ ✓

$\notin D$ (problem)

Bonus Solve for t: $A=P\left(1+\frac{r}{m}\right)^{mt}$.

$P\left(1+\frac{r}{m}\right)^{mt}=A$

$\left(1+\frac{r}{m}\right)^{mt}=\frac{A}{P}$

$\ln\left(\left(1+\frac{r}{m}\right)^{mt}\right)=\ln\left(\frac{A}{P}\right)$

$(\ln(1+\frac{r}{m}))\,mt=\ln(A/P)$

$$t=\frac{\ln(A/P)}{m\ln\left(1+\frac{r}{m}\right)}$$

10

5. Solve $2 \cdot 7^{x-1} - 3 = 0$ for x. Give an exact answer and then round to 4 decimal places. If you use this to supply the x-intercept for the appropriate graph on Page 1, it's worth a couple bonus points.

$2 \cdot 7^{x-1} - 3 = 0$ (10 pts)

$2 \cdot 7^{x-1} = 3$

$7^{x-1} = \frac{3}{2}$

$x - 1 = \log_7(3/2)$

$x = \log_7\left(\frac{3}{2}\right) + 1 = \frac{\ln\left(\frac{3}{2}\right)}{\ln(7)} + 1$

≈ 1.209367847

$\approx$ 1.2094 to 4 places

6. Solve $7^{x-3} = 5^x$ for x. Give an exact answer and then round your answer to 4 decimal places.

$\log_7(7^{x-3}) = \log_7(5^x)$

$x - 3 = (\log_7(5))x$

$x - \log_7(5)x = 3$

$x(1 - \log_7(5)) = 3$

$x = \frac{3}{1 - \log_7(5)}$

$= \frac{3}{1 - \frac{\ln 5}{\ln 7}}$ (10 pts)

≈ 17.34981318

$\approx$ 17.3498

20

7. Millsium has a half-life of 50 years, if I'm lucky. What's its decay rate?

$$Pe^{50k} = \frac{1}{2}P$$

$$e^{50k} = \frac{1}{2}$$

$$50k = \ln\left(\frac{1}{2}\right) = -\ln(2)$$

$$k = -\frac{\ln(2)}{50} \approx -.01386294436$$

10 pts

8. Using your work from the previous problem, a very old sample of radioactive ~~Weligmmium~~ millsium decayed from 20 grams to 3 grams. To the nearest *day*, how old is the sample?

$$20e^{kt} = 3$$

$$e^{kt} = \frac{3}{20}$$

$$kt = \ln\left(\frac{3}{20}\right)$$

$$t = \frac{\ln\left(\frac{3}{20}\right)}{k} =$$

$$= \frac{\ln\left(\frac{3}{20}\right)}{-\frac{\ln(2)}{50}} = \frac{-50\ln(3/20)}{\ln(2)} \approx 136.8482797$$

$$(.8482797 \text{ yr})\left(\frac{365 \text{ days}}{1 \text{ yr}}\right)$$

$$\approx 309.6220935$$

$$\approx 310 \text{ days}$$

136 yrs, 310 days

10 pts

10

9. Solve $(\log(x))^2 = \log(x^2)$ for x.

Let $u = \log(x)$

Then $\log(x^2) = 2\log(x) = 2u$ ✓

10 pts

$u^2 = 2u$

$u^2 - 2u = 0$

$u(u-2) = 0$

$u = 0$ or $u = 2$

$\log x = 0$

$x = 10^0 = 1$

$\log x = 2$

$x = 10^2 = 100$

$(\log(1))^2 = 0^2 = 0$ ✓

$\log(1^2) = \log(1) = 0$ ✓

$(\log(100))^2 = 2^2 = 4$ ✓

$\log(100^2) =$

$\log((10^2)^2) = \log(10^4) = 4$ ✓

10. What's the future value of \$5,000 invested at 7% APR, if interest is compounded... for 7 yrs

a. ... monthly?

$A = \$5000\left(1 + \frac{.07}{12}\right)^{12 \cdot 7} \approx \$8149.97$

8 pts

b. ... daily?

$A = \$5000\left(1 + \frac{.07}{365}\right)^{365 \cdot 7} \approx \$8161.20$

8 pts

c. ... continuously?

$A = \$5000e^{.07 \cdot 7} \approx \$8161.58$

5 pts