

9:28 - 9:50

This is our final learning opportunity together, and I'm hoping to take full advantage. Read the questions carefully. Sometimes, you can earn points on a problem by *knowing* that you did it wrong and *explaining* how you know and what you're *trying* to accomplish, and *how* you're going about it.

1. Solve the equation  $x^2 - 3x - 10 = 0$  in three different ways:

part a (15 pts) Factoring

$$(x-5)(x+2) = 0$$

$$x \in \{-2, 5\}$$

part b (15 pts) Completing the square

$$x^2 - 3x + \left(\frac{3}{2}\right)^2 = 10 + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{40 + 9}{4} = \frac{49}{4}$$

$$x - \frac{3}{2} = \pm \sqrt{\frac{49}{4}} = \pm \frac{7}{2}$$

$$x = \frac{3}{2} \pm \frac{7}{2}$$

$$\frac{3+7}{2} = \frac{10}{2} = 5$$

$$\frac{3-7}{2} = -\frac{4}{2} = -2$$

$$x \in \{-2, 5\}$$

part c (15 pts) Quadratic formula

$$a=1, b=-3, c=-10$$

$$b^2 - 4ac = (-3)^2 - 4(1)(-10)$$

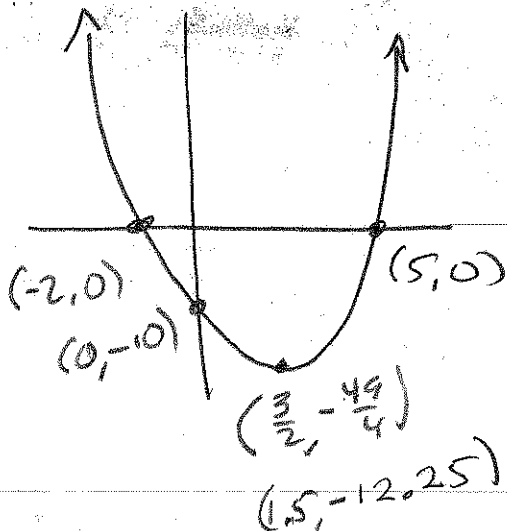
$$= 9 + 40 = 49$$

$$x = \frac{3 \pm \sqrt{49}}{2(1)} = \frac{3 \pm 7}{2} \begin{matrix} \nearrow +5 \\ \searrow -2 \end{matrix}$$

$$x \in \{-2, 5\}$$

2. (15 pts) Sketch the graph of  $f(x) = x^2 - 3x - 10$ . Include vertex, intercepts and be true to its shape. (No zig-zagging to make it go through calculated points.)

$$\left(x - \frac{3}{2}\right)^2 - \frac{49}{4}$$



3. Solve the absolute value inequality. Give your answer in set-builder *and* interval notation.

part a (10 pts)  $|7x + 2| < 4$

$$7x + 2 < 4 \text{ AND } 7x + 2 > -4$$

$$7x < 2$$

$$7x > -6$$

$$\left\{ x \mid x < \frac{2}{7} \text{ AND } x > -\frac{6}{7} \right\}$$

$$= \left(-\infty, \frac{2}{7}\right) \cap \left(-\frac{6}{7}, \infty\right)$$

$$= \left(-\frac{6}{7}, \frac{2}{7}\right)$$

part b (10 pts)  $|2x - 7| \geq 4$

$$2x - 7 \geq 4 \text{ OR } 2x - 7 \leq -4$$

$$2x \geq 11$$

$$2x \leq 3$$

$$\left\{ x \mid x \geq \frac{11}{2} \text{ OR } x \leq \frac{3}{2} \right\}$$

$$= \left(-\infty, \frac{3}{2}\right] \cup \left[\frac{11}{2}, \infty\right)$$

4. Let  $f(x) = \sqrt{x-18}$  and  $g(x) = x^2 + 3x - 10$

**part a** (10 pts) What's the domain of  $f(x)$ ? Give the answer in set-builder and interval notation.

$$\mathcal{D}(f) = \{x \mid x \geq 18\} = [18, \infty)$$

**part b** (10 pts) Determine  $(f \circ g)(x)$ . Simplify your answer.

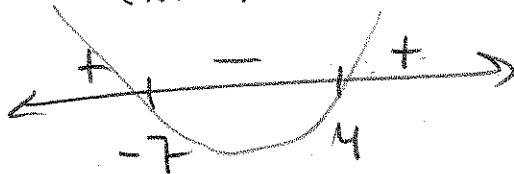
$$(f \circ g)(x) = \sqrt{x^2 + 3x - 10 - 18} = x^2 + 3x - 28$$

**part c** (10 pts) What's the domain of  $(f \circ g)(x)$ ? Give your answer in set-builder and interval notation.

$$\begin{aligned} & \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\} \\ & = \{x \mid x \in \mathbb{R} \text{ and } x^2 + 3x - 10 \geq 18\} \end{aligned}$$

$$\text{Need } x^2 + 3x - 28 \geq 0$$

$$(x+7)(x-4) \geq 0$$



$$\begin{aligned} \text{This gives } \mathcal{D} &= \{x \mid x \leq -7 \text{ OR } x \geq 4\} \\ &= (-\infty, -7] \cup [4, \infty) \end{aligned}$$

5. (10 pts) What is the domain of  $h(x) = \sqrt{(x-2)^3(x+1)(x-4)^2}$ ?

$$(x-2)^3(x+1)(x-4)^2 \geq 0$$

← + | - | + | + →  
          -1    2    4

$D = (-\infty, -1] \cup [2, \infty)$

6. (10 pts) What is the domain of  $\sqrt{\frac{(x-2)^3(x+1)}{(x-4)^2}}$ ? The hard part's done...

$$D = (-\infty, -1] \cup [2, 4) \cup (4, \infty)$$

7. (10 pts) Use synthetic division to find  $f(2)$  for  $f(x) = x^4 - 5x^3 - 3x^2 + 43x - 6$

$$\begin{array}{r|rrrrrr} 2 & 1 & -5 & -3 & 43 & -6 \\ & & 2 & -6 & -18 & 50 \\ \hline & 1 & -3 & -9 & 25 & 44 = f(2) \end{array}$$

8. (10 pts) Determine  $a$ ,  $r$  and  $n$  for the finite geometric series  $5 + 15 + 45 + \dots + 10935$

Then use  $a$ ,  $r$ , and  $n$  to determine the sum by the formula  $\sum_{k=1}^n a \cdot r^{k-1} = a \left( \frac{1-r^n}{1-r} \right)$ .

$$a=5, r=3$$

$$2187 = 3^{n-1} = 3^7$$

$$\Rightarrow n=8$$

$$5 \left( \frac{1-3^8}{1-3} \right) = 5 \left( \frac{1-6561}{-2} \right) = 5 \left( \frac{-6560}{-2} \right)$$

$$= 5(3280) = \boxed{16,400}$$

$$\begin{array}{r} 5 \overline{) 10935} \\ \underline{2187} \end{array}$$

9. (10 pts) How many ways can you pick 3 people from a group of 20 people to do a job for you? CHOOSE!

$$P(20, 3) = \frac{20!}{17!} = 20 \cdot 19 \cdot 18 = 6840$$

10. (10 pts) How many ways can you pick 3 people from a group of 20 people and then assign each of the 3 people to a different job? CHOOSE AND ARRANGE!

$$C(20, 3) = \frac{20!}{17! 3!} = \frac{20 \cdot 19 \cdot 18}{3 \cdot 2} = 20 \cdot 19 \cdot 3 = 1140$$

11. (10 pts) Use Pascal's Triangle (Binomial Theorem!) to help you expand  $(2x - 3)^5$ .

Expanding without using this technique will not earn any points.

$$\begin{aligned} & (2x)^5 + 5(2x)^4(-3) + 10(2x)^3(-3)^2 \\ & + 10(2x)^2(-3)^3 + 5(2x)(-3)^4 + (-3)^5 \\ & = 32x^5 - 240x^4 + 720x^3 - 1080x^2 + 810x - 243 \end{aligned}$$

		1		1			
		1	2	1			
		1	3	3	1		
		1	4	6	4	1	
		1	5	10	10	5	1

$$A = P \left( 1 + \frac{r}{m} \right)^{mt} = P(1+i)^n$$

$$FV = S = R \left( \frac{(1+i)^n - 1}{i} \right)$$

Previous semesters I used "FV" for future value of an annuity. I used 'S' this semester.

12. (10 pts) What's the future value, in 10 years, of \$10,000 deposited into a savings account, earning 4.3% annual percentage rate, compounded daily?

$$10000 \left( 1 + \frac{.043}{365} \right)^{365 \cdot 10} \approx \boxed{\$15,372.19}$$

.1859

13. (10 pts) An annuity consists of monthly payments of \$407 into an account earning 6% annual interest, compounded monthly, for 6 years. There are two ways to ask this question:

First way: How much does JG Wentworth feel that this annuity is worth? "I have a long-term settlement but I need cash NOW!"

Second way: If the annuity described is actually your monthly loan payments, how much did you borrow in the first place?

$$A = P(1+i)^n = R \left( \frac{(1+i)^n - 1}{i} \right) \Rightarrow$$

$$P = R \left( \frac{1 - (1+i)^{-n}}{i} \right) = 407 \left( \frac{1 - \left( 1 + \frac{.06}{12} \right)^{-(12)(6)}}{\left( \frac{.06}{12} \right)} \right)$$

$$\approx \boxed{\$24,558.18}$$

.18217

Check:  $(6)(12)(407) = 29,304$  is what you end up paying. Check:  $24558 \left( 1 + \frac{.06}{12} \right)^{6(12)} \approx 35,168.40$  ✓  
 (with no interest) ✓  $407 \left( \frac{(1 + \frac{.06}{12})^{12(6)} - 1}{\frac{.06}{12}} \right) \approx 35,168.40$  ✓