

1. Solve each equation. Identify each equation as an identity, inconsistent, or conditional equation.

a.  $\frac{1}{w-1} - \frac{1}{2w-2} = \frac{1}{2w-2}$

LCD =  $2(w-1)$

Domain:  $\{w \mid w \neq 1\}$

$$\frac{1}{w-1} - \frac{1}{2} = \frac{1}{2(w-1)} \Rightarrow$$

$$\frac{2-1}{2(w-1)} = \frac{1}{2(w-1)} \Rightarrow 1=1$$

IDENTITY.  
 $\{w \mid w \neq 1\}$  (or  $\mathbb{R} \setminus \{1\}$ )

or  $(-\infty, 1) \cup (1, \infty)$

b.  $\frac{z+2}{z-3} = \frac{5}{-3}$

LCD:  $-3(z-3)$

$$\left(\frac{z+2}{z-3}\right)\left(\frac{-3}{-3}\right) = \left(\frac{5}{-3}\right)\left(\frac{z-3}{z-3}\right)$$

$$\Rightarrow \frac{-3(z+2)}{-3(z-3)} = \frac{5(z-3)}{-3(z-3)}$$

$$\Rightarrow -3(z+2) = 5(z-3)$$

$$-3z - 6 = 5z - 15$$

$$-8z = -9$$

$$z = \frac{9}{8}$$

$$\{z \mid z = \frac{9}{8}\} \text{ or } z \in \left\{\frac{9}{8}\right\}$$

simply

$$z \in \left\{\frac{9}{8}\right\}$$

conditional

c.  $4 + \frac{6}{y-3} = \frac{2y}{y-3}$

LCD:  $y-3$

$$\left(\frac{4}{1}\right)\left(\frac{y-3}{y-3}\right) + \frac{6}{y-3} = \frac{2y}{y-3}$$

$$\Rightarrow 4(y-3) + 6 = 2y$$

$$\Rightarrow 4y - 12 + 6 = 2y$$

$$\Rightarrow 4y - 6 = 2y$$

$$2y = 6$$

$$y = 3$$

But  $y = 3 \notin \text{Domain!}$

So INCONSISTENT  
 Empty set =  $\emptyset = \{\} =$

2. Solve the absolute value equations

a.  $|x-4|=8$

$\Rightarrow x-4=8$  OR  $x-4=-8$

$\Rightarrow x=12$  OR  $x=-4$

$\Rightarrow x \in \{-4, 12\}$

b.  $|x+8|=-3$

$\Rightarrow \emptyset$  Never!

c.  $2|x+5|-10=0$

$\Rightarrow 2|x+5|=10$

$\Rightarrow |x+5|=5$

$\Rightarrow x+5=5$  OR  $x+5=-5$

$\Rightarrow x=0$  OR  $x=-10$

$\Rightarrow x \in \{-10, 0\}$

3. Solve  $(x+2)^2 = x^2 + 4$

$\Rightarrow x^2 + 4x + 4 = x^2 + 4$

$\Rightarrow 4x + 4 = 4$

$\Rightarrow 4x = 0$

$\Rightarrow x = 0$

$\Rightarrow x \in \{0\}$