Final Tuesday @ 10:10 to 12:00.

Recall geometric sum

$$
\begin{aligned}
bS' &= \cancel{ab} + \cancel{ab^2} + \cancel{ab^3} + \cdots + ab^n \\
-S' &= -(a + \cancel{ab} + \cancel{ab^2} + \cdots + \cancel{ab^{n-1}})
\end{aligned}
$$

$$bS' - S' = -a + ab^n = ab^n - a$$

$$S'(b-1) = a(b^n - 1)$$

$$S' = \frac{a(b^n-1)}{b-1}$$

$$2 + 2\cdot 3 + 2\cdot 3^2 + \cdots + 2\cdot 3^{23} \rightarrow n-1 = 23$$

$a = 2$

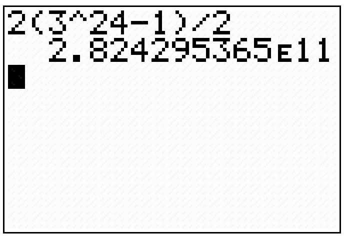
$b = 3$

$n = 24$

$$S' = \frac{2(3^{24}-1)}{3-1} \approx 2.824295365 \times 10^{11}$$

$= 282{,}429{,}536{,}500$

BIG!



Annuity - A stream of payments that earn interest.

Say $r = 8\%$ Payment = R
$t = 3$ yrs
$m = 12$ periods per year.

Payments always @ the end of the month.

$R(1+\frac{r}{m})^{mt-1}$ $= R(1+i)^{n-1}$
1st pmt.

Let $n = mt$ = total # of pmts.
$i = \frac{r}{m}$ = interest rate per month.

Future value of savings $A = P(1+\frac{r}{m})^{mt} = P(1+i)^n$ (for n periods).

$\underset{1^{st}}{R(1+i)^{n-1}} + \underset{2^{nd}}{R(1+i)^{n-2}} + \underset{3^{rd}}{R(1+i)^{n-3}} + \cdots + R(1+i)^2 + R(1+i)^1 + \underset{\text{Last pmt}}{R}$

$= R + R(1+i) + R(1+i)^2 + \cdots + R(1+i)^{n-1}$

Geometric Sum:
$a = R$
$b = (1+i)$
$n = n$

$i = \frac{r}{m}$
$n = mt$

Future Value of Annuity

$$\frac{a(b^n-1)}{b-1} = \boxed{\frac{R((1+i)^n-1)}{i} = S}$$

$1+i-1 = i$

THE Amortization Equation

The banker wants the annuity you promise him to have the same future value as if he'd put that loan in a bank and earned 8% compounded monthly on that deposit

$$A = P(1+i)^n$$

Say it's a loan of \$20,000

$$A = 20{,}000\left(1+\frac{.08}{12}\right)^{12\cdot 3} = \frac{R\left(\left(1+\frac{.08}{12}\right)^{12\cdot 3}-1\right)}{\frac{.08}{12}} = S'$$

$A = S'$ is what it's all about.

P = loan amt = present value of the annuity

R = monthly payments.

AMORTIZATION EQUATION. $A = S'$

$$P(1+i)^n = \frac{R\left((1+i)^n-1\right)}{i}$$

Depending on what's given, you use this to find payments or loan amt.

Do #s 1 & 2.

Nov 27-10:34 AM

#3 Saving for kids' college is a SINKING FUND question.

An annuity whose future value is known, but payments are unknown.

Want \$250,000 in 18 yrs.

$r = 5\%$ $\qquad n = 12 \cdot 18 = 216$

$t = 18$ yrs.

$$S = \frac{R((1+i)^n - 1)}{i} = 250000$$

$$\frac{R\left(\left(1+\frac{.05}{12}\right)^{216} - 1\right)}{\frac{.05}{12}} = 250{,}000$$

```
2(3^24-1)/2
  2.824295365E11
((1+.05/12)^(12*
18)-1)/(.05/12)
      349.2020215
```

```
2(3^24-1)/2
  2.824295365E11
((1+.05/12)^(12*
18)-1)/(.05/12)
      349.2020215
250000/Ans
      715.9179633
```

$349.2020215\, R \approx 250000$

$$R \approx \frac{250000}{349.2020215}$$

$\approx \$715.92$

Check:

$\frac{250000}{(12)(18)} \approx \underline{1157.41}$ if you earned NO interest, so \$715.92 makes sense.

$$* \quad A = S \quad *$$

$$P(1+i)^n = \frac{R((1+i)^n - 1)}{i}$$

I'll give you

$* \; A = P(1+i)^n$ &

$* \; S = \frac{R((1+i)^n - 1)}{i}$

$$\boxed{P = R\left(\frac{1-(1+i)^{-n}}{i}\right)}$$

Present Value of Annuity (Loan Amt)

$$\boxed{R = \frac{Pi}{1-(1+i)^{-n}}}$$

Payments for a given loan amt.

Sinking Fund.

Scratch: $S = \cancel{\frac{R((1+i)^n - 1)}{i}} \Longrightarrow$

$$\boxed{R = \frac{Si}{(1+i)^n - 1}}$$

Payments needed to save for college (or replace equipment)

Nov 27-10:55 AM