(9) $\sqrt{4^{2/\log_3(4)+2\log_9(3)}} = 6$, OBVIOUSLY

"Come sit by me," said the spider to the fly.

$$\sqrt{\sqrt[\log_3 4]{4^2}\cdot 4^{2\cdot\frac{1}{2}}}$$

$3=9^x \Rightarrow x=\frac{1}{2}$

$$=\sqrt[2]{\left(4^2\right)^{\frac{1}{\log_3 4}}\cdot 4}$$

Converting the exponent to something involving $\log_4$(stuff)

Scratch: $\frac{2}{\log_3 4} + 2\log_9(3)$

$\log_a x = \frac{\log_b x}{\log_b a}$

$$=\boxed{\frac{2}{\log_4(4)}\cdot\log_4(3)}+1$$

$$\frac{2}{\frac{\log_4 4}{\log_4 3}}$$

$$=\frac{2}{1}\log_4(3)+1$$

$$=\log_4(3^2)+1$$

This gives us

$$\sqrt{4^{\log_4(3^2)+1}}$$

$a^{b+c}=a^b a^c$

$$=\sqrt{4^{\log_4(3^2)}\cdot 4^1}$$

$$=\sqrt{3^2\cdot 4}=\sqrt{36}=6$$

Like #18, §8.2 Brute force:

$$\sum_{k=1}^{8}(k+5)(k-2) = (6)(-1)+(7)(0)+(8)(1)+(9)(2)+(10)(3)+(11)(4)+(12)(5)+(13)(6)$$

$k=1$

$(k+5)(k-2)$
$=(1+5)(1-2)$
$=(6)(-1)$

$$= -6+0+8+18+30+44+60+78$$
$$= 232$$

$$\sum_{k=1}^{8}(k^2+3k-10) = \sum_{k=1}^{8}k^2+\sum_{k=1}^{8}3k+\sum_{k=1}^{8}(-10)$$

I'll give you

$$\sum_{1}^{n}k = \frac{n(n+1)}{2}$$

$$\sum_{1}^{n}k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{1}^{n}k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{1}^{n}1 = n$$

$$= \sum_{k=1}^{8}k^2+3\sum_{k=1}^{8}k-10\sum_{k=1}^{8}1$$

$$= \frac{8(8+1)(2(8)+1)}{6}+3\,\frac{8(8+1)}{2}-10\cdot 8$$

$$= \frac{\overset{4}{\cancel{8}}(\overset{3}{\cancel{9}})(17)}{\underset{1}{\cancel{\underset{3}{\cancel{6}}}}}+\frac{\overset{12}{\cancel{24}}(9)}{\underset{1}{\cancel{2}}}-80$$

$$= 204+108-80$$
$$= 312-80$$
$$= 232$$

Brute force doesn't work on, say,

$$\sum_{k=1}^{1000}k = \frac{1000(1001)}{2} = 500\,(1001)$$
$$= 500500$$

$$\sum_{k=1}^{n} f(k) = \sum_{k=0}^{n-1} f(k+1)$$

$$f(1)+\dots+f(n) = f(1)+\dots+\underbrace{f(n-1+1)}_{f(n)}$$

$$\boxed{\sum_{k=0}^{n} f(k) = \sum_{k=1}^{n+1} f(k-1)}$$

To convert, manipulate the expression inside the $\sum$ so that 1st terms match. Check with last terms.

$$\underbrace{\sum_{k=0}^{50}(3k+1)}_{51 \text{ terms}} = \underbrace{\sum_{k=3}^{53} 3(k-3)+1}_{51 \text{ terms.}}$$

$k=0$: 1st: $3(0)+1$

$k=3$: 1st: $3(0)+1$

$0 = 3-3$
$= k-3$

$k=50$: $3(50)+1$

$k=53$: $3(53-3)+1$ ✓

Dealing with junk like

$$\sum_{k=0}^{50} k^2 = 0^2 + \underbrace{1^2 + \cdots + 50^2}_{\sum_{k=1}^{50} k^2 = \frac{50(51)(101)}{6}}$$

$$\sum_{k=-7}^{50} k^2 = (-7)^2 + (-6)^2 + (-5)^2 + \cdots + (-1)^2 + 0^2 + 1^2 + 2^2 + \cdots + 50^2$$

$$= \underbrace{7^2 + 6^2 + \cdots + 1^2}_{\sum_{k=1}^{7} k^2} + \cancel{0^2} + \underbrace{1^2 + 2^2 + \cdots + 50^2}_{\sum_{k=1}^{50} k^2}$$

$$\sum_{k=1}^{100} = \sum_{k=1}^{40} + \sum_{k=41}^{100}$$

$$\sum_{k=-5}^{50} k^3 = (-5)^3 + (-4)^3 + \cdots + (-1)^3 + 0^3 + 1^3 + \cdots + 50^3$$

$$= -5^3 - 4^3 - \cdots - 1^3 + 1^3 + \cdots + 50^3$$

Factor out -1 & re-order:

$$= -\left(1^3 + 2^3 + \cdots + 5^3\right) + 1^3 + 2^3 + \cdots + 50^3$$

$$= -\sum_{k=1}^{5} k^3 + \sum_{k=1}^{50} k^3$$

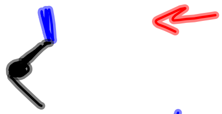
Find 1st 4 terms
§8.1 #54, sort of.

$a_n = a_{n-1} + 7$ $a_1 = -15$ = seed value

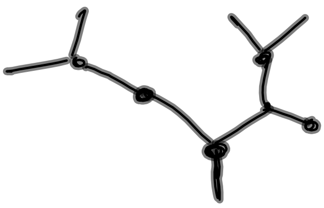
$a_2 = a_1 + 7 = -15 + 7 = -8$

$a_3 = a_2 + 7 = -8 + 7 = -1$

$a_4 = 7$



Fractals & computer graphics use recursions all the time!



GREAT DETAIL & small computer work.

Nov 7-10:47 AM

Geometric Series & Sequences.

Since r = Annual Percentage Rate, use the letter "b" for "base" when talking about geometric series

$$S_n = a + ab + ab^2 + \cdots + ab^{n-1}$$

$$= \frac{a(1-b^n)}{1-b}$$

Practice Test #3 Monday b4 Thanksgiving.

Practice Test #4 Wednesday, next week.