$$\frac{-q-8}{(q+3)} < 0 \qquad \frac{(q+8)}{q+3} > 0$$

critical values:

$-q-8=0$ $\quad q+3=0$

$-q=8$ $\quad q=-3$

$q=-8$

critical: $q=-8$, $q=-3$

Rosa says the horizontal asymptote can help you build your sign pattern.

H.A.: $\frac{-q-8}{q+3} \xrightarrow{|q|\to\infty} \frac{-q}{q} = \boxed{-1=y}$ is H.A.

$-$ $\quad +$ $\quad -$

-8 $\quad -3$

$(-\infty,-8)\cup(-3,\infty)$

Nice job, Rosa.

$$\frac{-(-7)-8}{-7+3} = \frac{-1}{-4} = \frac{1}{4} + \checkmark$$

(

Recall

$$\sum_{k=1}^{5} k = 1+2+3+4+5 = 15$$ Easy Brute Force

$$= \text{Gauss says} = \frac{5(5+1)}{2} = \frac{30}{2}$$

$$\sum_{k=1}^{120} k = 1+2+3+\cdots+120$$ Hard Brute Force.

$$= \frac{120(121)}{2} = \frac{n(n+1)}{2} = 7260 = \frac{n^2+n}{2} = \boxed{\frac{n^2}{2} + \text{smaller}}$$

For integral calculus

$$\sum_{k=1}^{5} k^2 = 1^2+2^2+3^2+4^2+5^2$$

$$= 1+4+9+16+25 = 55$$

$$= \frac{5(5+1)(5(2)+1)}{6} = \frac{5\cancel{(6)}(11)}{\cancel{6}} = 55$$

$$\begin{array}{r} 33 \\ 5 \\ \hline 165 \end{array}$$

$$= \frac{n(n+1)(2n+1)}{6} = \frac{2n^3 + \text{smaller}}{6} = \boxed{\frac{n^3}{3} \cancel{+ \text{smaller}}}$$

$$\sum_{k=1}^{5} k^3 = 1^3+2^3+3^3+4^3+5^3$$

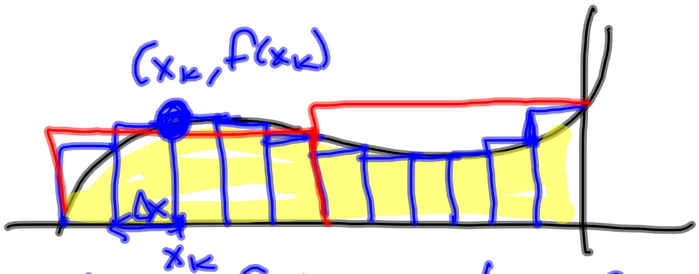
$$= 1+8+27+64+125$$

$$= 225$$

$$= \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^2(n+1)(n+1)}{4} = \frac{n^4 + \text{smaller}}{4}$$

$$= \boxed{\frac{n^4}{4} + \text{smaller}}$$

Preview of Calculus.



Area of the rectangles ≈ Area under the curve.

More rectangles ⟹ Better Estimate

Newton says:
Use infinitely many rectangles ⟹
Get the exact area!

limit as # of rectangles goes to infinity!

Sum of the areas =
rect$_1$ + rect$_2$ + rect$_3$ + ... + rect$_n$

$= f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \cdots + f(x_n)\Delta x$

$= \sum_{k=1}^{n} f(x_k)\Delta x \approx$ Area

This is where $\sum k$, $\sum k^2$, $\sum k^3$ comes up.

Area under $f(x) = x^2$
$\approx \sum_{k=1}^{n} k^2 \Delta x$

$\xrightarrow{n \to \infty} \int x^2 dx =$ Area

$\frac{n^3}{3} + \cancel{\text{smaller}}$ $\qquad = \frac{x^3}{3}$

$\sum k \quad \sum k^2 \quad \sum k^3$

1, 3, 9, 27, 81, ... is a geometric sequence

Start @ $\boxed{k=1}$

$a_1 = a = 1$

$a_2 = 3a = 3 = 3 \cdot a_1$

$a_3 = 9a = 3^2 a = 3 \cdot a_2$

$a_4 = 27a = 3^3 a = 3 \cdot a_3$

$\vdots$

$a_n = a_{n-1} \cdot 3$

$a_0 = a = 1^{st}$ term

r = "common ratio"

In general,

$\boxed{a_n = a r^{n-1}}$

Geometric series / sum

$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$

$= 7 + 21 + 63 + \cdots + 7 \cdot 3^{n-1}$

$= 7 + 7 \cdot 3 + 7 \cdot 3^2 + \cdots + 7 \cdot 3^{n-1}$

Find a formula for S_n

$S_n = a + ar + ar^2 + \dots + ar^{n-1} = \sum_{k=1}^{n} ar^{k-1}$

$- \quad rS_n = ar + ar^2 + ar^2 + \dots + ar^n$

$S_n - rS_n = a - ar^n$

$S_n(1-r) = a - ar^n$ $\qquad x - rx = x(1-r)$

$S_n = \frac{a - ar^n}{1-r} = \frac{a(1-r^n)}{1-r}$

$1250 \times 5 = 6250$

1 2 3 4 5 6

$2 + 10 + 50 + 250 + 1250 + 6250 = 7812$

It's geometric, with $a = 2$

$r = 5$

$n = 6$

$\frac{a(1-r^n)}{1-r} = \frac{2(1-5^6)}{1-5} = 7812$

$2 + 10 + 50 + \dots + 488281250$

$a = 2$

$r = 5$

$n =$

$\sum_{k=1}^{n} ar^{n-1}$

$= 2 + 10 + 50 + \dots + 2 \cdot 5^{12}$

$n = 13$

$\frac{2(1-5^{13})}{1-5}$

$= 610351562$

5 | 488281250
5 | 97656250
5 | 19531250
5 | 3906250
5 | 781250
5 | 156250
5 | 31250
5 | 6250
5 | 1250
5 | 250
5 | 50
5 | 10
2

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