

Solve

$$\pi^x = 3^{x+1}$$

$$\log_{\pi}(\pi^x) = \log_{\pi}(3^{x+1})$$

$$x = (x+1) \log_{\pi}(3)$$

$$\text{Let } b = \log_{\pi}(3)$$

$$x = (x+1)b = bx + b$$

$$x - bx = b$$

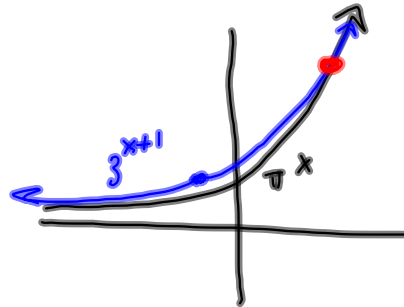
$$x(1-b) = b$$

$$x = \frac{b}{1-b} = \frac{\log_{\pi}(3)}{1 - \log_{\pi}(3)} \approx \frac{7}{10}$$

$$\frac{8}{10} = \frac{\frac{\ln(3)}{\ln(\pi)}}{1 - \frac{\ln(3)}{\ln(\pi)}} = \frac{\ln(3)/\ln(\pi)}{(1 - \ln(3)/\ln(\pi))}$$

To 4 places:  
 $\frac{\ln(3)/\ln(\pi)}{\ln(\pi)} \approx 23.8220$

|   |
|---|
| $\sqrt{\ln(81)}$                        |
| $1/\sqrt{2}$                            |
| $\ln(3)/\ln(\pi)$                       |
| $\ln(3)/\ln(\pi)/(1 - \ln(3)/\ln(\pi))$ |
| $\ln(3)/\ln(\pi)$                       |
| 23.8219759                              |



$$x = (x+1)(5)$$

$$x = 5x + 5$$

$$-5x = -5x$$

$$-4x = 5$$

$$x = -\frac{5}{4}$$

$$\pi^x = 3^{x+1}$$

$$\log_3(\pi^x) = \log_3(3^{x+1})$$

$$x \log_3 \pi = x+1$$

$$x \log_3 \pi - x = 1$$

$$x(\log_3 \pi - 1) = 1$$

$$x = \frac{1}{\log_3 \pi - 1} = \frac{1}{\frac{\log \pi}{\log 3} - 1} = \frac{1}{\frac{\ln \pi}{\ln 3} - 1} \approx$$

```

.7071067812
ln(3)/ln(pi)/(1-1
n(3)/ln(pi))
23.8219759
1/(ln(pi)/ln(3)-1
)
23.8219759

```

23.8220

$$3^{x+1} = \pi^x$$

$$\ln(3^{x+1}) = \ln(\pi^x)$$

$$(x+1)\ln 3 = x\ln \pi$$

$$x\ln 3 + \ln 3 = x\ln \pi$$

$$(\ln 3)x + \ln 3 = (\ln \pi)x$$

$$(\ln 3)x - (\ln \pi)x = -\ln 3$$

$$(\ln 3 - \ln \pi)x = -\ln 3$$

$$x = \frac{-\ln 3}{\ln 3 - \ln \pi}$$

|                       |            |
|-----------------------|------------|
|                       | 23.8219759 |
| 1/(ln(pi)/ln(3)-1)    |            |
| }                     | 23.8219759 |
| -ln(3)/(ln(3)-ln(pi)) |            |
| }                     | 23.8219759 |

## Recall Annuity

Payments are made into a bank account at the beginning of the month for 4 years. The account earns 4% APR, compounded monthly. How much will you have @ maturity? **Make pmts of \$400**

$$i = \frac{r}{m} = \frac{.04}{12}$$

$$\left. \begin{array}{l} t = 4 \text{ yrs} \\ m = 12 \text{ per year} \\ r = .04 \end{array} \right\} n = mt = 48$$

$$a + ab + ab^2 + \dots + ab^{n-1} = \frac{a(1-b^n)}{1-b}$$

$$R(1+i)^n + R(1+i)^{n-1} + \dots + R(1+i)^2 + R(1+i)$$

$$\underline{R(1+i)} + \underline{R(1+i)^2} + \dots + \underline{R(1+i)^{n-1}} + \underline{R(1+i)^n}$$

$$\downarrow \quad \downarrow$$

$$a \quad R(1+i)(1+i) + \dots + R(1+i)(1+i)^{n-2} + R(1+i)(1+i)^{n-1}$$

$$a = R(1+i)$$

$$b = (1+i)$$

$$n = 48$$

$$R = 400$$

$$So \quad \sum_{48} = \frac{a(1-b^n)}{1-b}$$

$$= \frac{R(1+i)(1 - (1+i)^{-48})}{1 - (1+i)}$$

$$= \frac{400(1 + \frac{.04}{12})(1 - (1 + \frac{.04}{12})^{-48})}{- \frac{.04}{12}}$$

$$400 * (1 + \frac{.04}{12}) * (1 - (1 + \frac{.04}{12})^{-48}) / (-.04/12)$$

|                 |             |
|-----------------|-------------|
| 400*48          | 19200       |
| 400*(1+.04/12)* |             |
| 1-(1+.04/12)^48 |             |
| /(-.04/12)      |             |
|                 | 20853.11986 |

$$\approx 20,853.12$$

Same Problem. Pmts @ END of month

$$R(1+i)^{n-1} + R(1+i)^{n-2} + \dots + R(1+i) + R$$

1<sup>st</sup> pmt earns  
n-1 periods' worth  
of interest.

$$= R + R(1+i) + \dots + R(1+i)^{n-1}$$

$$= \frac{R(1 - (1+i)^n)}{-i}$$

Simple, ordinary  
annuity certain

$$1 - (1+i) = -i$$

Loans: Banker is buying an annuity from you.

He expects you to earn him interest for the money you borrowed

$P$  = Principal       $R$  = Payments

$$A = \textcircled{P}(1+i)^n = R \frac{(1-(1+i)^n)}{-i} = \text{FV of annuity}$$

The loan equation.

$$= \frac{R((1+i)^n - 1)}{i}$$