#104 Add part (e) Continuous compounding - $A = Pe^{rt}$

(4.1 #s 50, 56, 104) Like worksheet #9

4.2 #s 36, 38, 44, 46, 132, 134

§4.3 #s 50, 52, 62, 68, 70, 76, 78, 84, 98*

*Do 2 versions:

(1) Mattie says $A = P(1+r)^t$ (compounded annually) Book

(2) I say $A = Pe^{rt}$ (compounded continuously) Convenient

version (2) should be a slightly smaller r-value.

§4.4 #s 44, 48, 54, 76

$\hookrightarrow$ K-Ar

For #48: $a^b a^c = a^{b+c}$

$$3^{x^2-5} \cdot 3^{2x} \cdot 3^{27}$$

$$= 3^{x^2-5+2x+27} = 3^{x^2+2x+22}$$

Back to §4.3 for a smidge.

Like #98 – See note about $P(1+r)^t$ and Pe^{rt} methods.

#97

Maverick (1970, 1995)

(2009, 10,890)

Use the compound interest formula to find annual INFLATION rate.

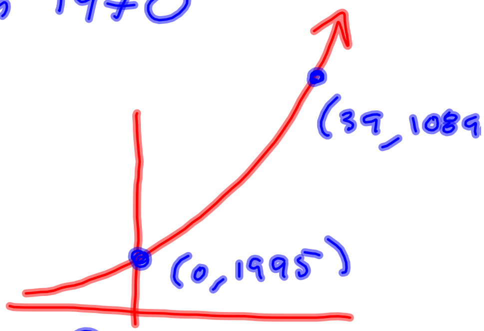
Book wants $A=P(1+r)^t$

Let t = time, in years after 1970

(0, 1995)

(39, 10890)

$$\begin{array}{r} 2009 \\ -1970 \\ \hline 39 \end{array}$$



① $1995(1+r)^t$ OR

② $1995e^{rt}$

① $A = P\boxed{(1+r)}^t = A(t)$

$= 1995(1+r)^t$

And $A(39) = 1995(1+r)^{39} = 10890.$

Solve for r.

Take 39^{th} root, at some point.

② $A = Pe^{rt}$

$= 1995e^{rt} = A(t)$

$A(39) = 1995e^{39r} = 10890$

Solve for r

§4.4 #5 ... Let's do some examples like the homework.

44 – 40 or fight

44 is like #43

Like #48

$$3^{x} \cdot 3^{x+1} = 9^{x^2+x}$$

$$3^{x+x+1} = 9^{x^2+x}$$

$$3^{2x+1} = 9^{x^2+x}$$

SLEDGEHAMMER

Variable in the exponent: logs bring 'em down.

$$\ln(3^{2x+1}) = \ln(9^{x^2+x})$$

$$(2x+1)\ln 3 = (x^2+x)\ln 9$$

Let $a = \ln 3$, $b = \ln 9$

$$(2x+1)a = (x^2+x)b$$

$$2ax + a = bx^2 + bx$$

$$bx^2 + bx - 2ax - a = 0$$

Oct 31-10:16 AM

$bx^2 + (b-2a)x - a = 0$

I use a's & b's in quadratic formula.

Relabel: $a=m$, $b=w$

$wx^2 + (w-2m)x - m = 0$ $\quad (a-b)^2 = a^2-2ab+b^2$

$a=w$, $b=w-2m$, $c=-m$

$$b^2-4ac = (w-2m)^2 - 4(w)(-m)$$
$$= w^2 - 2(w)(2m) + (2m)^2 + 4wm$$
$$= w^2 - 4wm + 4wm + 4m^2$$
$$= w^2 + 4m^2$$

$$x = \frac{-(w-2m) \pm \sqrt{w^2+4m^2}}{2w}$$
$$= \frac{2m-w \pm \sqrt{w^2+4m^2}}{2w}$$
$$= \frac{2a-b \pm \sqrt{b^2+4a^2}}{2b}$$

$$4(\ln 3)^2 = 2^2(\ln(3))^2 = (2\ln(3))^2 = (\ln(3^2))^2 = (\ln(9))^2$$

$$= \frac{2\ln 3 - \ln(9) \pm \sqrt{(\ln 9)^2 + 4(\ln 3)^2}}{2\ln 9}$$

owie! But it should be right

Here's my boo-boo

$$= \frac{\ln(3^2) - \ln(9) \pm \sqrt{\ln(9)^2 + \ln(9)^2}}{\ln(81)}$$
$$= \frac{\pm\sqrt{2\ln(9)^2}}{\ln(81)}$$

```
  -1.847319574
-log(e^(1))/(log
(e^(1))-1)
   .7677041641
√(ln(9)+ln(9)^2)
/ln(81)
±  .6031416942
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Real World hurts.

$$= \pm\frac{\ln 9\sqrt{2}}{\ln(81)}$$
$$= \pm\frac{\ln 9\sqrt{2}}{2\ln 9} \qquad \ln(81) = \ln(9^2) = 2\ln 9$$
$$= \pm\frac{\sqrt{2}}{2}$$ Yes!

$$3^x \cdot 3^{x+1} = 9^{x^2+x}$$

Now, this is a cooked-up problem. You SHOULD exploit the $3^2 = 9$ thing

$$3^{2x+1} = (3^2)^{x^2+x}$$

$$3^{2x+1} = 3^{2x^2+2x} \quad 3^{2(x^2+x)}$$

$$2x+1 = 2x^2+2x$$

exponentials are 1-to-1.

$a^b = a^c \Longleftrightarrow b = c$

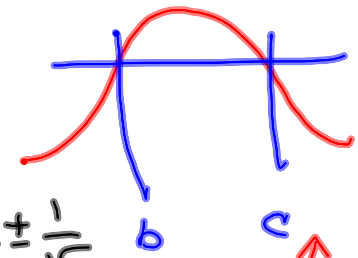
$$2x^2+2x-2x-1 = 0$$

$$2x^2-1 = 0$$

$$2x^2 = 1$$

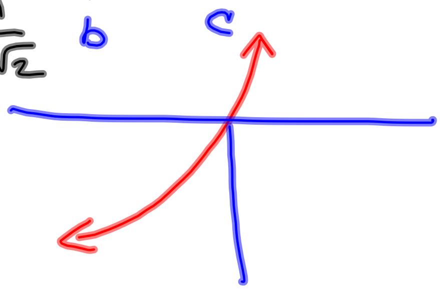
$$x^2 = \frac{1}{2}$$

$$x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{1}}{\sqrt{2}} = \pm\frac{1}{\sqrt{2}}$$



NOT 1-to-1, so $b = c$ may not happen

$$\frac{1}{1.4142} \approx \frac{1}{\sqrt{2}}$$



$b = c$

$1.4142 \overline{)1.000000}$

There's a mistake in my sledgehammer page. I'll find it.

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$2\overset{.7071}{\overline{)1.4142}}$$

Much easier long division

To me, $\frac{1}{\sqrt{2}}$ is just as good as $\frac{\sqrt{2}}{2}$

$$3^x \cdot 3^{x+1} = 9^{x^2+x}$$

$$3^{2x+1} = 9^{x^2+x}$$ cheap trick

$$(2x+1)\ln 3 = (x^2+x)\ln 9 = (x^2+x)\ln(3^2) = (x^2+x)(2\ln 3)$$

$$(2x+1)\cancel{\ln(3)} = 2(x^2+x)\cancel{\ln(3)}$$

But assume we're not that clever.
Let $m = \ln 3$, $w = \ln 9$. Then

$$m(2x+1) = w(x^2+x) \longrightarrow$$

$$2mx + m = wx^2 + wx \longrightarrow$$

$$wx^2 + wx - 2mx - m = 0 \longrightarrow$$

$$wx^2 + (w-2m)x - m = 0 \longrightarrow$$

$$a = w,\ b = w-2m,\ c = -m \longrightarrow$$

$$b^2 - 4ac = (w-2m)^2 - 4(w)(-m)$$

$$= w^2 - 4wm + 4m^2 + 4wm$$

$$= w^2 + 4m^2 \longrightarrow$$

$$x = \frac{-(w-2m) \pm \sqrt{w^2+4m^2}}{2w}$$

$$= \frac{2m - w \pm \sqrt{w^2+4m^2}}{2w}$$

$$= \frac{2\ln 3 - \ln 9 \pm \sqrt{(\ln 9)^2 + \cdots}}{\cdots}$$

This is what I missed the 1st time