Quizzes/Other:	40%
Quiz	20%
Practice Test	20%
midterm	30%
Final	30%

Oct 19-10:05 AM

§3.6 Rational Functions $\frac{P(x)}{Q(x)}$

zeros $f(x)=0$ (Numerator $=0$) Find x

y-int $f(0)$

Domain $Q(x)\neq 0$

vertical Asymptotes $\boxed{Q(x)=0 \text{ \& } P(x)\neq 0}$

Horizontal Asymptotes $|x|\to$ BIG

Proper & Improper Functions

Obliqu Asymptotes.

Improper

$Q(x)=0$ AND $P(x)=0$

HOLE

Graph $\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)}$

It's <u>Proper</u>

$\frac{1}{3}$ is proper

$\frac{4}{3}$ is improper

$\frac{x}{x^2+2}$, $\frac{1}{x^2-1}$, $\frac{3x^3+7x}{5x^4-7x^3+29x}$ are proper

$\frac{x^2+2}{x}$ is improper (Degree of denominator is greater than the degree of the numerator.

When <u>PROPER</u>,

$\boxed{\text{horizontal asymptote } y=0}$

(Denominator Dominates)

From last time:

$f(x) = \frac{1}{(x-1)(x+1)}$

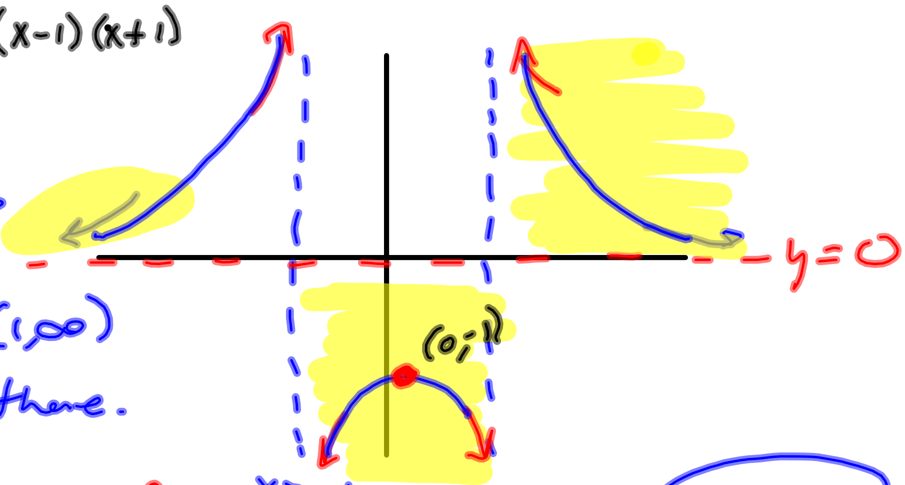
x-intercepts: <u>NONE</u> $1 \neq 0$

$\mathcal{D} = \{x \mid x \neq \pm 1\}$

$-1 \quad 1$

$= (-\infty,-1) \cup (-1,1) \cup (1,\infty)$

Numerator $\neq 0$ there.



Test		f(x)			
$(-\infty,-1)$	$x=-2$	$\frac{1}{(-2-1)(-2+1)}$	$=\frac{1}{(-3)(-1)}$	$=\frac{1}{3}$	+
$(-1,1)$	$x=0$	$\frac{1}{(0-1)(0+1)}$	$=\frac{1}{(-1)(1)}$	$=-1$	−
$(1,\infty)$	$x=2$	$\frac{1}{(2-1)(2+1)}$	$=\frac{1}{(1)(3)}$	$=\frac{1}{3}$	+

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Horizontal Asymptotes

$\frac{x}{x^3+2}$, $\frac{1}{x^2+27x}$, ... Proper $\Rightarrow y=0$

$$\frac{2x^3+5x^2-1}{7x^3-4x+25}$$

$y=\frac{2}{7}$

$$\frac{2x^3+5x^7-2x+1}{6x^7-2200x^3-10^6x+11}$$

$y=\frac{5}{6}$

when $|x|$ is BIG, the high powers Dominate.

$\xrightarrow{|x|\to\infty}$ $\frac{2x^3}{7x^3}=\frac{2}{7}$

$$\frac{5x^7+\text{smaller}}{6x^7+\text{smaller}} \xrightarrow{|x|\longmapsto} \frac{5x^7}{6x^7}=\frac{5}{6}$$

$$R(x) = \frac{2x^2 \cancel{-7x-15}}{3x^2 \cancel{+5x+2}} = \frac{(2x+3)(x-5)}{(3x+2)(x+1)}$$

$(2x+3)(x-5)$
$= 2x^2 - 7x - 15$

$(3x+2)(x+1)$
$= 3x^2 + 5x + 2$

x-int: $\left(-\frac{3}{2}, 0\right), (5, 0)$

y-int: $\left(0, -\frac{15}{2}\right)$

$\mathcal{D} = \{x \mid x \neq -\frac{2}{3} \text{ AND } x \neq -1\}$

Are these zeros of the numerator?

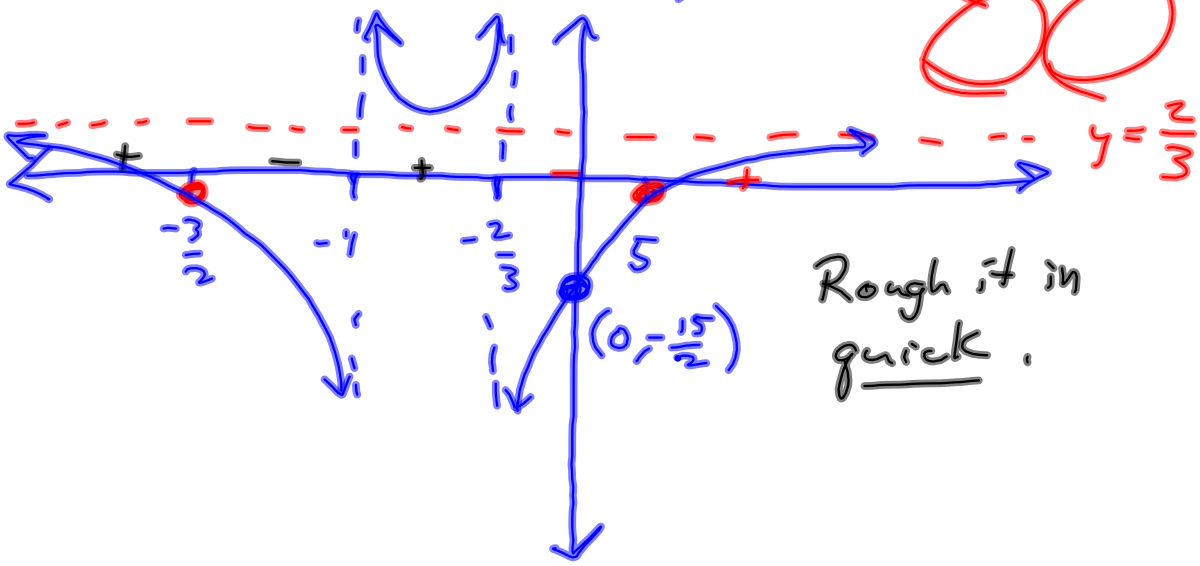
No $\Longrightarrow$ Vertical Asymptotes

H.A.: $y = \frac{2}{3}$ $\quad \frac{2x^2 + \text{small}}{3x^2 + \text{small}} \xrightarrow{|x| \to \infty} \frac{2}{3}$

$$= \frac{(2x+3)^1 (x-5)^1}{(3x+2)^1 (x+1)^1}$$

$x = -3/2, 5, -\frac{2}{3}, -1$ are where it can change sign.

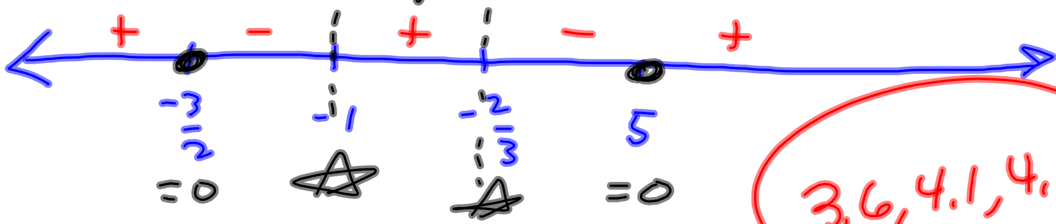
$-\frac{3}{2}, -1, -\frac{2}{3}, 5$



Rough it in quick.

Solve $\dfrac{(2x+3)(x-5)}{(3x+2)(x+1)} > 0$

Sign pattern:



$(-\infty, -\frac{3}{2}) \cup (-1, -\frac{2}{3}) \cup (5, \infty)$

3.6, 4.1, 4.2

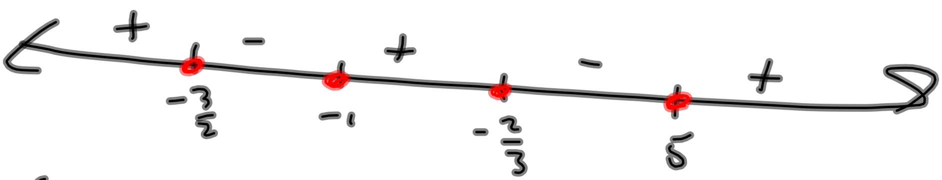
Next time.

$\dfrac{(2x+3)(x-5)}{(3x+2)(x+1)} \geq 0$

$(-\infty, -\frac{3}{2}] \cup (-1, -\frac{2}{3}) \cup [5, \infty)$

Fine point: Domain plays a role.

$(2x+3)(x-5)(3x+2)(x+1) \geq 0$



$(-\infty, -\frac{3}{2}] \cup [-1, -\frac{2}{3}] \cup [5, \infty)$