`:= x→sort(expand((2·x − 3)·(x + 3)·(x − 2 + sqrt(2)·I)·(x − 2 − sqrt(2)·I)))

$x \to sort(expand((2x-3)(x+3)(x-2+I\sqrt{2})(x-2-I\sqrt{2})))$

`(x)

$2x^4 - 5x^3 - 9x^2 + 54x - 54$

Find all real zeros & factor over the reals.
Each ~~time~~ you find a zero, split off a linear factor $x-c$. Then work with the ~~bummed-out~~ depressed polynomial.

$x=c$

2	54
3	27
3	9
	3

Descartes: 3 or 1 pos. zeros

$f(-x) = 2x^4+5x^3-9x^2-54x-54$

1 neg. zero.

$\frac{p}{q}$: $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18, \pm27, \pm54$
$\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{27}{2}$ are the possible rational zeros

$2x^4-5x^3-9x^2+54x-54$

$$\begin{array}{r|rrrrr} 1 & 2 & -5 & -9 & 54 & -54 \\ & & 2 & -3 & -12 & 42 \\ \hline & 2 & -3 & -12 & 42 & \end{array}$$
NewP

$$\begin{array}{r|rrrrr} -1 & 2 & -5 & -9 & 54 & -54 \\ & & -2 & 7 & 2 & \\ \hline & 2 & -7 & -2 & 56 & \end{array}$$
Newp

$$\begin{array}{r|rrrrr} 2 & 2 & -5 & -9 & 54 & -54 \\ & & 4 & -2 & -22 & \\ \hline & 2 & -1 & -11 & 32 & \end{array}$$
Newp

$$\begin{array}{r|rrrrr} -2 & 2 & -5 & -9 & 54 & -54 \\ & & -4 & 18 & -18 & -72 \\ \hline & 2 & -9 & 9 & 36 & \end{array}$$
NewP.

$$\begin{array}{r|rrrrr} 3 & 2 & -5 & -9 & 54 & -54 \\ & & 6 & 3 & -18 & \\ \hline & 2 & 1 & -6 & 36 & \end{array}$$
NewP.

$$\begin{array}{r|rrrrr} -3 & 2 & -5 & -9 & 54 & -54 \\ & & -6 & 33 & -72 & +54 \\ \hline -3 & 2 & -11 & 24 & -18 & 0 \\ & & -6 & 51 & -\text{Huge} & \\ \hline & 2 & -17 & 75 & -\text{Huge} & \end{array}$$

$f(x) = \underline{(x+3)}(2x^3-11x^2+24x-18)$

$x=-3$ is Lower Bound on real zeros

$\frac{1}{2}$ Didn't work

$$\begin{array}{r|rrrr} \frac{3}{2} & 2 & -11 & 24 & -18 \\ & & +3 & -12 & 18 \\ \hline & 2 & -8 & 12 & 0 \end{array}$$
Sweet!

$f(x) = (x+3)(x-\frac{3}{2})(2x^2-8x+12)$

style only.
$= (x+3)(x-\frac{3}{2})(2)(x^2-4x+6)$
$= (x+3)(2x-3)(x^2-4x+6)$

Now solve $2x^2-8x+12=0$
$2(x^2-4x+6)=0$
$x^2-4x+6=0$

$x^2-4x+2^2 = -6+4$
$(x-2)^2 = -2$
$x-2 = \pm\sqrt{-2} = \pm i\sqrt{2}$
$x = 2\pm i\sqrt{2}$

RIGHT HERE is where I realize this has no real zeros.

So, $f(x) = 2(x+3)(x-\frac{3}{2})(x-(2+i\sqrt{2}))(x-(2-i\sqrt{2}))$

To finish the question asked,
$f(x) = (x+3)(x-3/2)(2x^2-8x+12)$
is Done factoring over the reals.
The "$2x^2-8x+12$" is an "irreducible quadratic factor."

Oct 16-9:13 AM

$$f(x) = (x+3)(x-\tfrac{3}{2})\overbrace{(2x^2-8x+12)}^{(-8)^2-4(2)(12)\,=\,64-96\,=\,-32\,<\,0}$$

No real...

When you get to this point, the quick & slick analyst does the discriminant for $2x^2-8x+12=0$

$$2(x^2-4x+6)=0$$

$$b^2-4ac = (-4)^2-4(1)(6)$$
$$= 16-24$$
$$= -8 \quad \text{No real zeros!}$$

Followup question: Find ALL zeros. We did that, already, 'cuz I got ahead of myself.

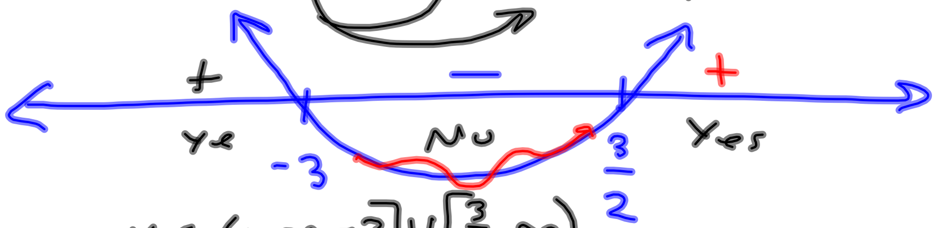
Linear factors

$$f(x) = 2(x+3)(x-\tfrac{3}{2})(x-(2+i\sqrt{2}))(x-(2-i\sqrt{2}))$$

$$x = -3,\ \tfrac{3}{2},\ 2\pm i\sqrt{2}$$

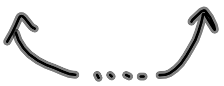
$f(x) = 2(x+3)^1(x-\tfrac{3}{2})^1(x-(2+i\sqrt{2}))(x-(2-i\sqrt{2}))$

Solve $f(x) \geq 0$ want "+"

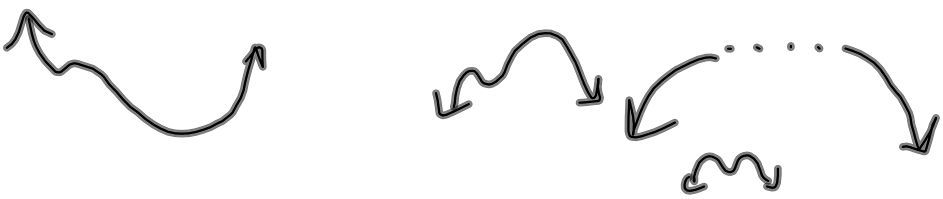


$x \in (-\infty,-3] \cup [\tfrac{3}{2},\infty)$

$f(x) = 2x^4 +$ smaller. (See pg 313)

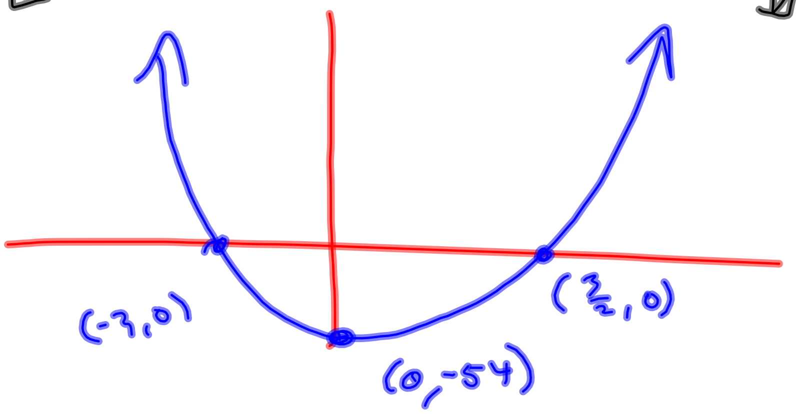


$-2x^4 + \ldots.$



$7x^3 + \ldots.$

$-7x^3$



GOOD ENOUGH

Role of nonreal zeros in the graph:

None.

3.4 #s 8, 12, 22, 30, 36, 42

3.5 #s 72, 74, 78, 81, 82

After you graph them, solve the inequalities!

#72 $f(x) > 0$

#74 $f(x) \geq 0$

#78 $f(x) < 0$

#82 $f(x) \leq 0$

Oct 16-10:14 AM