The Fundamental Theorem of Algebra

If y = P(x) is a polynomial function of positive degree, then y = P(x) has at least one zero in the set of complex numbers.

And if we use the factor theorem to "split off the linear factor (x - c) corresponding to one of the zeros," we get a *depressed polynomial*, of one degree less than P(x)'s degree. We then apply FTA to the depressed polynomial, use the factor theorem (and synthetic division) to split off another linear factor, until we've split off *every* linear factor, and a splitting of P(x) into linear factors is achieved.

In class, last time, (and time before) I told you that FTA says that a polynomial of degree *n* has *n* complex zeros. And that's what always having at least *one* means, once you have the Factor Theorem under your belt!

$$P(x) = a_{n}x^{n} + a_{n}x^{n-1} + \dots + a_{1}x + a_{0}$$

$$= a_{n}(x - c_{1}) \cdot (x - c_{2}) \cdot (x - c_{3}) \cdots (x - c_{n})$$

$$P(x) = 3x^{2} - x - 2 \qquad (3x+2)(x-1)$$

$$= 3x^{2} - x - 2 = 0$$

$$P(x) = 3(x - (-\frac{2}{3}))(x - 1) \implies x = -\frac{2}{3}, 1$$

$$= 3(x - c_{1})(x - c_{2})$$

$$P(x) = 10(x + 1)(x - \frac{2}{3})(x + \frac{2}{3})$$

Example build for Rational Zeros Theorem and Descartes' Rule of Signs. These "educate" our guesses at what the zeros *might* be. We check our guesses with synthetic division and when we guess right, we've found a zero *and* we've lowered the degree of the polynomial we're working with by one degree!

with by one degree!

$$2 \cdot x + 3 = P(x)$$
 $2 \cdot x + 3$
 $2 \cdot (x + 1)$
 $2 \cdot (x + 1)$

Note:

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Theorem on Bounds

Suppose that P(x) is a polynomial with real coefficients and a positive leading coefficient, and synthetic division with x - c is performed.

- If c > 0 and all terms in the bottom row are nonnegative, then c is an upper bound for the roots of P(x) = 0.
- If c < 0 and the terms in the bottom row alternate in sign, then c is a lower bound for the roots of P(x) = 0.

3.3 #65 is a good example for this theorem.

P(x)= $10 \times^3 + 21 \times^3 + 5 \times -6$ We saw an upper bound example, already. Now for a lower bound: $x=-1,-\frac{3}{2},\frac{2}{5}$ Try x=-3 $x=-1,-\frac{3}{2},\frac{2}{5}$ $x=-1,-\frac{3}{2},\frac{2}{5}$

Conjugate Pairs Theorem

If P(x) = 0 is a polynomial equation with real coefficients and the complex number a + bi ($b \ne 0$) is a root, then a - bi is also a root.

I'm not so sure this is a really practical tool for breaking down a polynomial, in practice. But it's GREAT for building a polynomial in factored form that turns out to have real coefficients when you expand it. I use this theorem all the time, when I'm building examples for class. Also, it's really easy to write test questions for this concept!

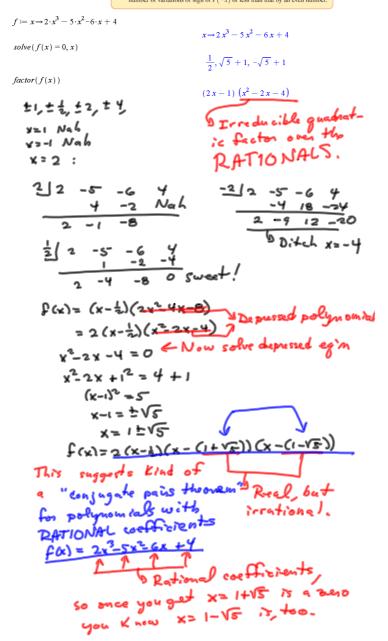
expand(%)

$$(x-2+3\cdot I)(x-2-3\cdot I)\cdot (3\cdot x-2)\cdot (x-1)$$
 $(x-2+3\cdot I)(x-2-3\cdot I)(3x-2)(x-1)$
 $(x-3)(x-2+3\cdot I)(x-2-3\cdot I)(x-2-3\cdot I)(3x-2)(x-1)$
 $(x-3)(x-2+3\cdot I)(x-2-3\cdot I)(x-2-3\cdot I)(3x-2)(x-1)$
 $(x-3)(x-2+3\cdot I)(x-2-3\cdot I)(x-2-3\cdot I)(3x-2)(x-1)$
 $(x-3)(x-2+3\cdot I)(x-2-3\cdot I)(x$

Descartes's Rule of Signs

Suppose P(x) = 0 is a polynomial equation with real coefficients and with terms written in descending order.

■ The number of positive real roots of the equation is either equal to the number of variations of sign of P(x) or less than that by an even number.
■ The number of negative real roots of the equation is either equal to the number of variations of sign of P(-x) or less than that by an even number.



Descartes's Rule of Signs

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- The number of negative real roots of the equation is either equal to the number of variations of sign of P(-x) or less than that by an even number.

$$f := x \to 2 \cdot x^3 - 5 \cdot x^2 - 6 \cdot x + 4$$

$$x \to 2 x^3 - 5 x^2 - 6 x + 4$$

$$solve(f(x) = 0, x)$$

$$\frac{1}{2}, \sqrt{5} + 1, -\sqrt{5} + 1$$

$$factor(f(x))$$

$$(2x - 1) (x^2 - 2x - 4)$$

 $f(x)=2x^{3}-5x^{2}-6x+4$ positive zeros $f(-x)=2(-x)^{3}-5(-x)^{2}-6(-x)+4$ $=-2x^{3}-5x^{2}+6x+4$ I sign change.

Exactly 1 negative zero.

3.3 #5 15, 16, 18 [23, 26, 28, 62, 67]
Midterm

On tests, you will

On tests, you will

NOT examd these Types

one from C3 of questions

FRIDAY.