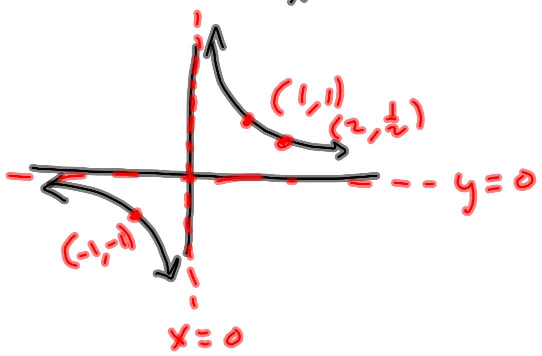
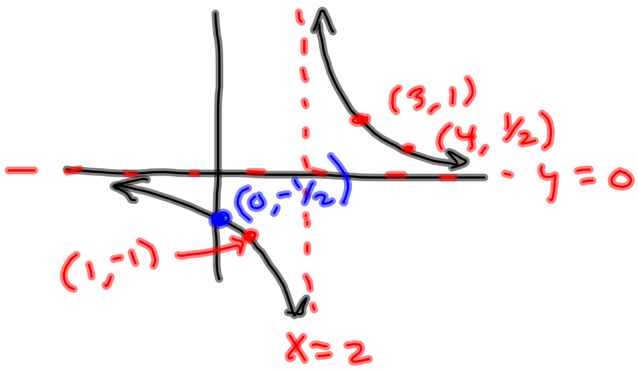
$$h(x) = \frac{1}{x-2} + 3$$

$$f(x) = \frac{1}{x}$$

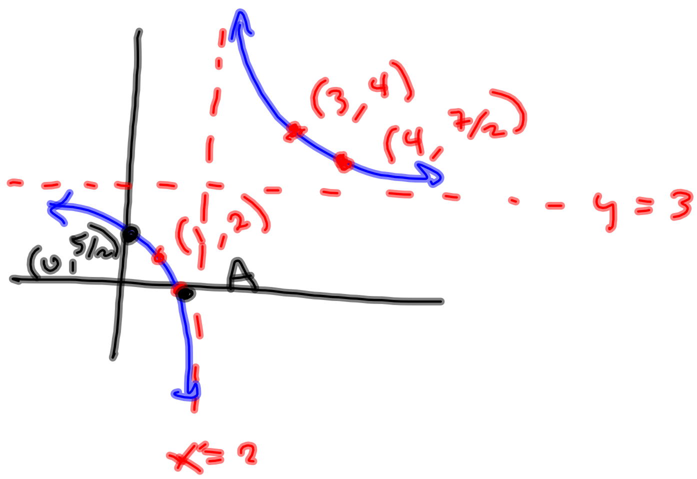


$$f(x-2) = \frac{1}{x-2}$$



$$f(x-2) + 3 = h(x) = \frac{1}{x-2} + 3$$

$$3 + \frac{1}{2} = \frac{7}{2}$$



A: $h(x) = 0$

$$\frac{1}{x-2} + 3 = 0$$

$$\frac{1}{x-2} + \frac{3(x-2)}{x-2} = 0$$

$$\frac{1+3x-6}{x-2} = 0$$

$$-5 + 3x = 0$$

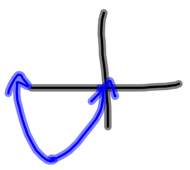
$$3x = 5$$

$$x = \frac{5}{3}$$

$$\boxed{A = (5/3, 0)}$$

$y = \boxed{f(x) = x^2 + 4x}$

$= x^2 + 4x + 2^2 - 4$ $\quad a(x-h)^2 + k$



$= (x+2)^2 - 4$

$(h, k) = (-2, -4)$

y-int: $(0,0)$ $x = 0$

x-int: $y = 0$

$x^2 + 4x = 0$

$(x+2)^2 - 4 = 0$

$x(x+4) = 0$

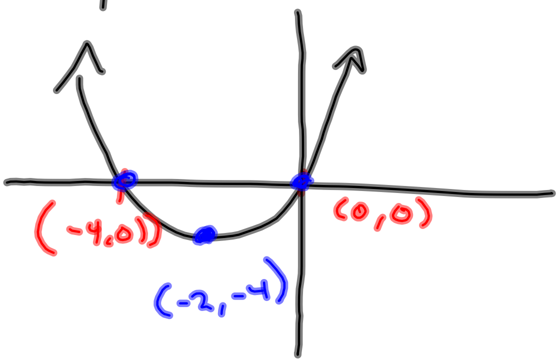
$(x+2)^2 = 4$

$x = 0$ OR $x + 4 = 0$

$x + 2 = \pm\sqrt{4} = \pm 2$

$\boxed{x = -4}$

$x = -2 \pm 2 \rightarrow (0,0),\ (-4,0)$



$3x^2-4x-4 \le 0$

$(3)(-4)=-12$

want factors
to sum to -4

$(-6)(2)=-12$

$-6+2=-4$

$3x^2-6x+2x-4 \le 0$

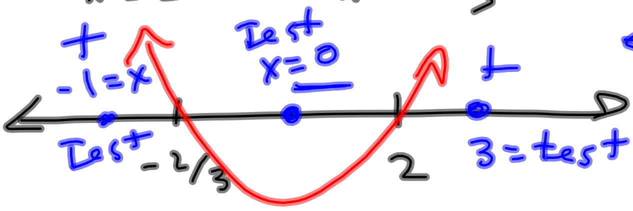
$3x(x-2)+2(x-2) \le 0$

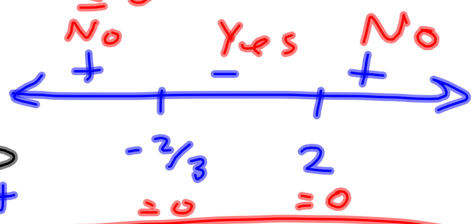
$(x-2)(3x+2) \le 0$

Critical:

$x-2=0$ $\quad 3x+2=0$

$x=2$ $\quad x=-2/3$





$\boxed{[-2/3, 2]}$

$(x-2)(3x+2)$

Test: $x=3$

$(3-2)(3(3)+2)$ $\quad +$

Test $x=0$:

$(0-2)(3(0)+2)$ $\quad -$

Test $x=-1$

$(-1-2)(3(-1)+2)$

$(-3)(-1)$ $\quad +$

$3x^2 - 4x - 4 \le 0$

$a = 3, b = -4, c = -4$

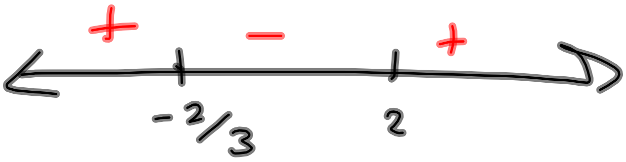
$b^2 - 4ac = (-4)^2 - 4(3)(-4)$

$= 16 + 48$

$= 64$

$x = \dfrac{4 \pm \sqrt{64}}{2(3)} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{4 \pm 8}{6} \to 2, \; -\dfrac{2}{3}$



etc.

Can we now factor this? Yes. The zeros give the factors:

$3x^2 - 4x - 4$

$= 3(x-2)(x+\frac{2}{3})$

$= (x-2)(3)(x+2/3)$

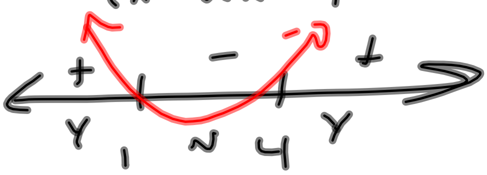
$\boxed{= (x-2)(3x+2)}$

$5x - x^2 < 4$

$-x^2 + 5x - 4 < 0$

$x^2 - 5x + 4 > 0$

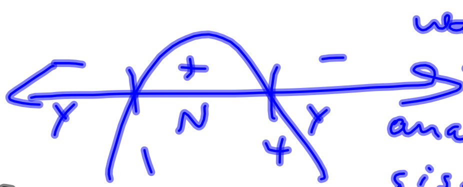
$(x-1)(x-4) > 0$



>0 analysis

$-(x^2 - 5x + 4) < 0$

$-(x-4)(x-1) < 0$



want <0 analysis.

Gives

$x \in (-\infty, 1) \cup (4, \infty)$

$f(x) = x^4 + x^3 - 11x^2 + 13x + 1$

Divide: $f(x) \div (x-2)$

$$\begin{array}{r|rrrrr} 2 & 1 & 1 & -11 & 13 & 1 \\ & & 2 & 6 & -10 & 6 \\ \hline & 1 & 3 & -5 & 3 & \boxed{7 = r = f(2)} \\ & x^3 & x^2 & x & c & \end{array}$$

Answer to part b.

$f(x) = (x-2)(x^3 + 3x^2 - 5x + 3) + 7$

(2) $f(2) = 2^4 + 2^3 - 11(2)^2 + 13(2) + 1$

$= 16 + 8 - 44 + 26 + 1$

$= 7$

REMAINDER THEOREM

f(2) = remainder upon division by x - 2.

FACTOR THEOREM

Remainder Theorem when $r=0$

$f(c)=0 \iff x-c$ is a factor

Example Build

$(x+3)(x^2-5x+7)$

x^3-5x^2+7x

$3x^2-15x+21$

$f(x)=x^3-2x^2-8x+2$

Another

$(x+3)(x^2+4x+5)$

x^3+4x^2+5x

$3x^2+12x+15$

$g(x)=x^3+7x^2+17x+15$

Find *all* zeros of f(x) and sketch it.

$f(x) = x^3 - 2x^2 - 8x + 21$ ($x = -3$ is a zero)

(Divide by $x+3$)

$-3 + 3 = 0$

$$\begin{array}{r|rrrr} -3 & 1 & -2 & -8 & 21 \\ & & -3 & 15 & -21 \\ \hline & 1 & -5 & 7 & 0 \end{array}$$

$f(x) = (x+3)(x^2 - 5x + 7)$

Solve the "depressed equation

$x^2 - 5x + 7 = 0$

$a = 1, b = -5, c = 7$

$b^2 - 4ac = (-5)^2 - 4(1)(7)$

$= 25 - 28 = -3$

So $x = \frac{-b \pm \sqrt{-3}}{2a} = \frac{5 \pm i\sqrt{3}}{2}$ and,

$f(x) = (x+3)\left(x - \left(\frac{5+\sqrt{3}i}{2}\right)\right)\left(x - \left(\frac{5 - i\sqrt{3}}{2}\right)\right)$

zeros $x = -3, \frac{5 \pm i\sqrt{3}}{2}$

One real zero, two nonreal zeros.

Fundamental Theorem of Algebra.

Rational Zeros Theorem

Descartes' rule of signs Theorem

Bounds on Real Zeros.

Oct 8-10:52 AM