3 ways to solve a quadratic equation

(1) Quadratic Formula

(2) Completing the Square

(3) Factoring

Solve $2x^2-5x+3=0$

$$2x^2-3x-2x+3=0$$

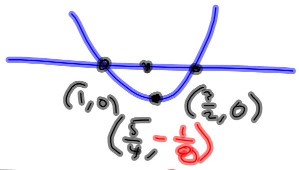
$$x(2x-3)-(2x-3)=0$$

$$(2x-3)\left(\frac{x\cancel{(2x-3)}}{\cancel{2x-3}}-\frac{\cancel{2x-3}}{\cancel{2x-3}}\right)=0$$

$$(2x-3)(x-1)=0$$

$2x-3=0$ or $x-1=0$

$x=\frac{3}{2}$ or $x=1$



$$f\left(\tfrac{5}{4}\right)=2\left(\tfrac{5}{4}\right)^2-5\left(\tfrac{5}{4}\right)+3$$

$$=\frac{2(25)}{16}-\frac{25}{4}+3$$

$$=\frac{25}{8}-\frac{50}{8}+\frac{24}{8}$$

$$=-\frac{1}{8}$$

$$2x^2-5x+3=2\left(x^2-\tfrac{5}{2}x\qquad\right)+3$$

$2\cdot\frac{25}{16}=\frac{25}{8}$ $\qquad=2\left(x^2-\frac{5}{2}x+\left(\frac{5}{4}\right)^2\right)+3-\frac{25}{8}$

$$=2\left(x-\tfrac{5}{4}\right)^2-\frac{1}{8}$$

Solve for x-int: Read it: Vertex is @ $\left(\frac{5}{4},-\frac{1}{8}\right)$

$$2\left(x-\tfrac{5}{4}\right)^2-\frac{1}{8}=0$$

$$2\left(x-\tfrac{5}{4}\right)^2=\frac{1}{8}$$

$$\left(x-\tfrac{5}{4}\right)^2=\frac{1}{16}$$

$$x-\frac{5}{4}=\pm\frac{1}{4}$$

$$x=\frac{5}{4}\pm\frac{1}{4}\;\begin{cases}\to \frac{6}{4}=\frac{3}{2}\\ \to 1\end{cases}$$

This tells us where (0,0) went for $f(x)=x^2$

$(0,0)\longrightarrow(0,0)$

$x^2\longrightarrow 2x^2$

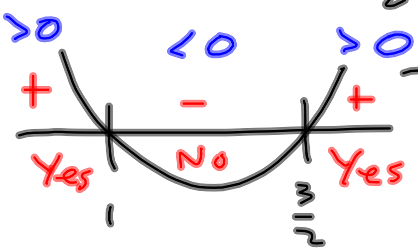
$\longrightarrow 2\left(x-\frac{5}{4}\right)^2 \quad \left(\frac{5}{4},0\right)$

$\longrightarrow 2\left(x-\frac{5}{4}\right)^2-\frac{1}{8} \quad \left(\frac{5}{4},-\frac{1}{8}\right)$

Solve $2x^2 - 5x + 3 \ge 0$

$(2x-3)(x-1) \ge 0$

$x = \frac{3}{2}, 1$ are "critical"

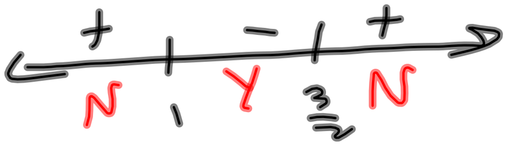


$\Rightarrow x \in (-\infty, 1] \cup [\frac{3}{2}, \infty)$

Sign Pattern for the function

Solve $2x^2 - 5x + 3 \le 0$

Same sign pattern. Different yes/no



$x \in [1, \frac{3}{2}]$

$\frac{3278}{9}$ $3278 \div 9$

$$
\begin{array}{r}
364\ r\ 2 \\
9\ \overline{)\ 3278} \\
-2700 \\
\hline
578 \\
-540 \\
\hline
38 \\
36 \\
\hline
2
\end{array}
$$

Interpret:

$$\frac{3278}{9} = 364 + \frac{2}{9}$$

$$3278 = \underbrace{9 \cdot 364}_{3276} + 2$$

$3276 + 2$

$$\frac{x^3+2x^2-6x+8}{x-2}$$

$$
\begin{array}{r l}
 & x^2+4x+2 \quad r\ 12 \\
x-2 \,\big) & x^3+2x^2-6x+8 \\
 & +\ (-x^3+2x^2) \\
\hline
 & 4x^2-6x+8 \\
 & +\ (-4x^2+8x) \\
\hline
 & 2x+8 \\
 & -(2x-4) \\
\hline
 & 12
\end{array}
$$

$$\frac{x^3}{x} = x^2$$

$$x^2(x-2)$$
$$= x^3-2x^2$$
$$4x\,(x-2)$$
$$= 4x^2-8x$$

This says

$$\frac{x^3+2x^2-6x+8}{x-2} = x^2+4x+2+\frac{12}{x-2}$$

$$f(x) = x^3+2x^2-6x+8 = (x-2)(x^2+4x+2)+12$$

$$f(2) = 2^3+2(2)^2-6(2)+8$$
$$= 16-12+8 = 12$$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -6 & 8 \\ & & 2 & 8 & 4 \\ \hline & 1 & 4 & 2 & 12 \end{array}$$

$\rightarrow f(2) = 12$

We divided

$\boxed{x^3 + 2x^2 - 6x + 8 \text{ by } x - 2}$

Remainder Theorem
When polynomial $f(x)$ is divided by $x - c$, then $f(c)$ is the remainder.

Factor Theorem:
If remainder is zero, then $x - c$ is a FACTOR of $f(x)$ and $f(c) = 0$

FIND ALL ZEROS !

$g(x) = x^3 + 2x^2 - 6x - 4$ by $x - 2$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -6 & -4 \\ & & 2 & 8 & 4 \\ \hline & 1 & 4 & 2 & 0 \\ & x^2 & x^1 & c & r \end{array}$$

This says:

$x^3 + 2x^2 - 6x - 4 = (x-2)(x^2 + 4x + 2)$; so,

to find the remaining zeros, solve

$$x^2 + 4x + 2 = 0$$

$$x^2 + 4x = -2$$

$$x^2 + 4x + 2^2 = -2 + 4$$

$$(x+2)^2 = 2$$

$$x + 2 = \pm\sqrt{2}$$

$$x = -2 \pm \sqrt{2}$$

zeros are $x = 2, -2 \pm \sqrt{2}$