2.5 #s 5-8, 11-14, 16-30, 49efgh, 50efgh,
51-62, 69-74, 83, 84 $\rightarrow |x| \neq x$

2.6 #s 1-18, 25-28, 37-40

S'2.4 #s 37-50

$f(x) = 3x - 1$, $g(x) = x^2 + 1$, $h(x) = \frac{x+1}{3}$

37 $f(g(-1))$

$= f((-1)^2 + 1)$

$= f(2)$

$= 3(2) - 1$

$= 5$

fog in gen'l

$f(g(x)) =$

$f(x^2+1) =$

$3(x^2+1) - 1$

$= 3x^2 + 3 - 1$

$= 3x^2 + 2 \Longrightarrow$

$f(g(-1)) = 3(-1)^2 + 2$

$= 3 + 2$

$= 5$

$f(x) = 3x-1$

$$f \circ f = 3(3x-1)-1$$
$$= f(f(x))$$
$$= f(3x-1)$$

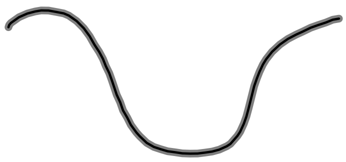
1-to-1 §2.5

Crucial for Ch 4 Exponentials & Logarithms are inverse functions.

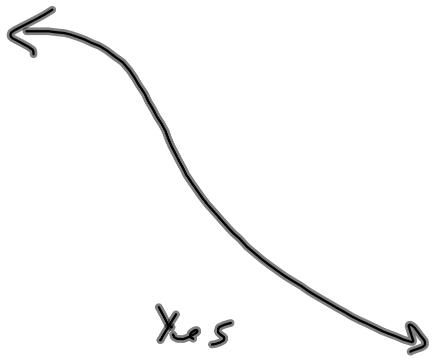
function:

x corresponds to one y Vertical Line

1-to-1 function is a function and y corresponds to one x. Horizontal Line test.

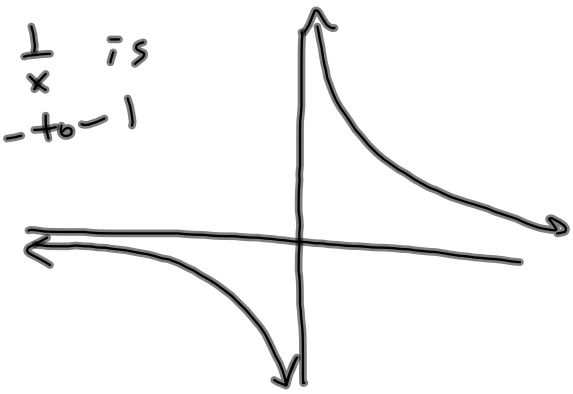


Func. Not 1-to-1



Yes

$\frac{1}{x}$ is 1-to-1



1-to-1 functions have INVERSES that are functions.

The inverse of f, denoted f^{-1}

Not a -1 power but a Function inverse.

It's an inverse with respect to function composition.

Multiplicative Identity

1

$1 \cdot 72 = 72$

$3 \cdot \frac{1}{3} = 1$

Function identity

$f(x) = x$

f^{-1} composed with f gives the identity function

$f^{-1} \circ f = x$

$f(x) = 3x \Longrightarrow f^{-1}(x) = \frac{1}{3}x$

$f^{-1}(f(x)) = f^{-1}(3x) = \frac{1}{3}(3x) = x$ back home.

$f(x) = \frac{x^3+5}{7}$ Find f^{-1} by reversing a composition.

what does f do to x?

1. $x \to x^3$ cube
2. x^3+5 add 5
3. $\frac{x^3+5}{7}$ divide by 7

3. cube root $\sqrt[3]{7x-5} = f^{-1}(x)$
2. subtract 5 $7x-5$
1. Multiply by 7 $7x$

$$f(f^{-1}(x)) = \frac{\left(\sqrt[3]{7x-5}\right)^3 + 5}{7}$$

$$= \frac{7x-5+5}{7} = \frac{7x}{7} = x$$

$$f^{-1}(f(x)) = \sqrt[3]{7\left(\frac{x^3+5}{7}\right)-5}$$

$$= \sqrt[3]{x^3+5-5} = \sqrt[3]{x^3} = x$$

$y = \frac{x^3+5}{7} = f(x)$

more general tool:

substitute/swap x & y. Solve for y.

$\frac{y^3+5}{7} = x$

$y^3+5 = 7x$

$y^3 = 7x-5$

$y = \sqrt[3]{7x-5}$

Sep 25-10:36 AM

$$f(x) = \frac{x-3}{x+2} = y$$

$$\frac{y-3}{y+2} = (x)\left(\frac{y+2}{y+2}\right)$$

$$\frac{y-3}{y+2} = \frac{x(y+2)}{y+2}$$

$$y-3 = x(y+2)$$

$$y = xy + 2x + 3$$

$$y - xy = 2x+3$$

$$y(1-x) = 2x+3$$

$$y = \frac{2x+3}{1-x} = f^{-1}(x)$$

Sep 25-10:39 AM

$$f(x) = \frac{x-3}{x+2} = y \qquad y = \frac{2x+3}{1-x} = f^{-1}(x)$$

check:

$$f^{-1}(f(x)) = \frac{2f+3}{1-f} \quad (\text{to "see"})$$

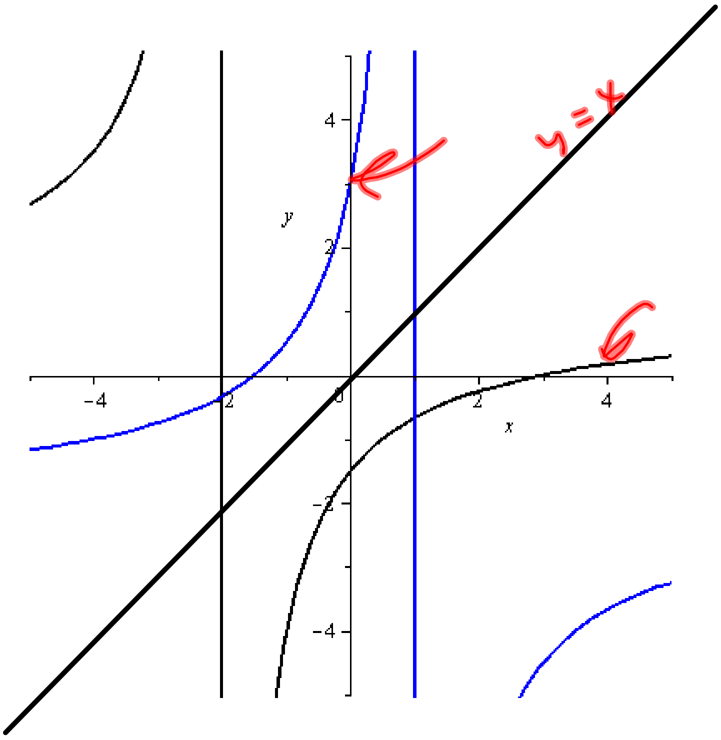
$$= \frac{2\left(\frac{x-3}{x+2}\right)+3}{1-\frac{x-3}{x+2}} = \frac{\frac{2(x-3)}{x+2} + \frac{3(x+2)}{x+2}}{\frac{1(x+2)}{x+2} - \frac{x-3}{x+2}}$$

$$\frac{x+2-(x-3)}{x+2} = \frac{x+2-x+3}{x+2}$$

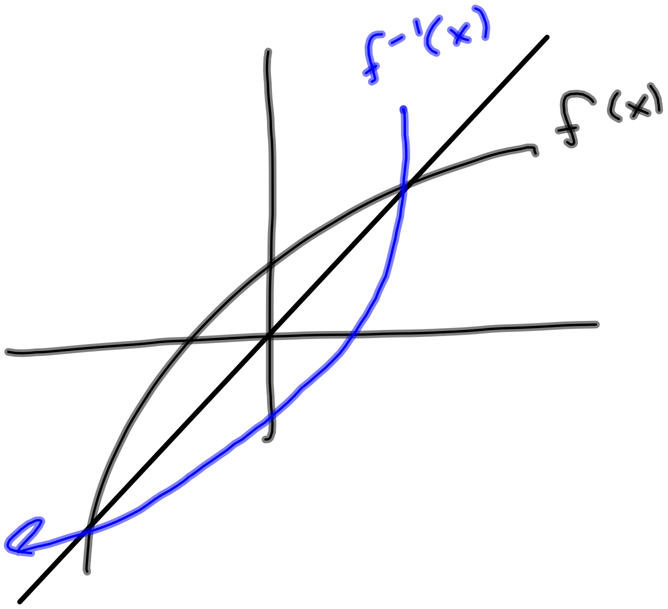
$$= \frac{\frac{2x-6+3x+6}{x+2}}{\frac{x+2-x+3}{x+2}}$$

$$= \frac{\frac{5x}{x+2}}{\frac{5}{x+2}} = \frac{5x}{x+2} \cdot \frac{x+2}{5} = x$$

Maple
CAS.



Mirror
images in
y=x



Sep 25-10:52 AM

Two methods :

① Reverse Composition (Limited)

② $\underbrace{\text{Swap } x \,\&\, y \,\&\, \text{solve}}_{\text{Switch \& solve (Book calls it.)}}$ for y (Sledgehammer)

Motivation: Need this idea for solving logarithmic and exponential equations. CHAPTER 4

$$3^{x^2+2} = 1$$

$$\log_3\left(3^{x^2+2}\right) = \log_3(1) = 0$$

$$x^2+2 = 0, \text{etc}$$

Sep 25-10:54 AM