$f(x) = 3x + 1 \implies$

f(☺) = 3☺ + 1

$f(7x-11) = 3(7x-11) + 1$

$= 21x - 33 + 1$

$= 21x - 32$

$f(r) = 3r + 1$

$f(x) = 2x^2 + 5x - 2$

$f(x+h) = 2(x+h)^2 + 5(x+h) - 2$

$= 2(x^2 + 2xh + h^2) + 5x + 5h - 2$

$= 2x^2 + 4xh + 2h^2 + 5x + 5h - 2$

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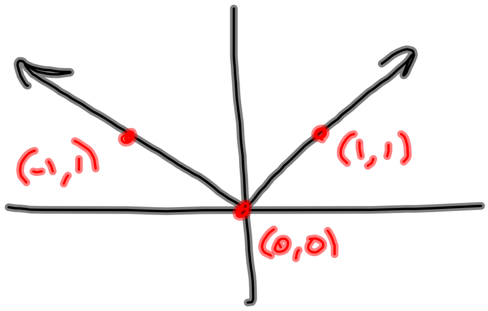
Examples of functions (families)

Func.	Basic Func.
① $g(x) = ax+b$ linear function	$f(x) = x$ identity function
② $g(x) = ax^2+bx+c$ quadratic function	$f(x) = x^2$ squaring function square ..
③ $g(x) = ax^3+bx^2+cx+d$ cubic function	$f(x) = x^3$ cube function cubing ..
④ $g(x) = a\sqrt{bx+c} + d$ square root function	$f(x) = \sqrt{x}$ square root function
⑤ $g(x) = a\sqrt[3]{bx+c} + d$	$f(x) = \sqrt[3]{x}$ cube root function
⑥ $g(x) = a\lvert bx+c \rvert + d$	$f(x) = \lvert x \rvert$ absolute value.
⑦ $g(x) = \frac{a}{bx-c}$ Reciprocal	$f(x) = \frac{1}{x}$
⑧ $g(x) = \frac{a}{(bx-c)^2}$ Reciprocal squared?	$f(x) = \frac{1}{x^2}$

⑨ $f(x) = \lfloor x \rfloor$

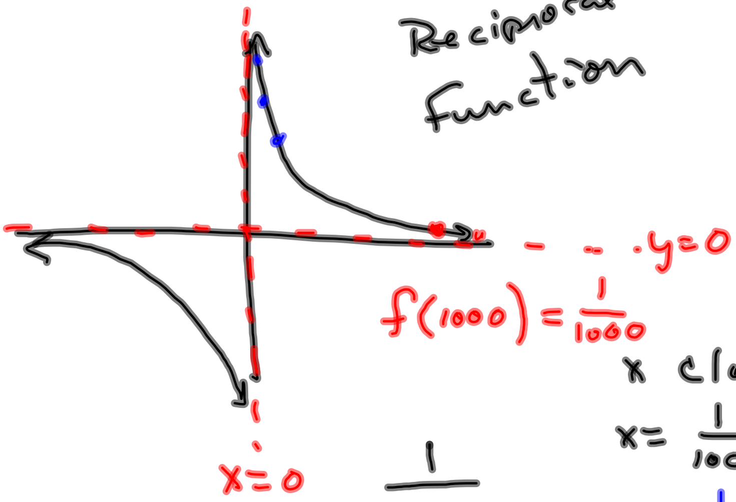
Greatest integer (less than or equal to x)

⑥ $f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$ piecewise defin.



⑦ $f(x) = \frac{1}{x}$

Reciprocal Function



x	y
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
0	$\cancel{\square}$
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$

$\frac{1}{0}$

$f(1000) = \frac{1}{1000}$

x close to zero:

$x = \frac{1}{100} \Longrightarrow f(x) = 100$

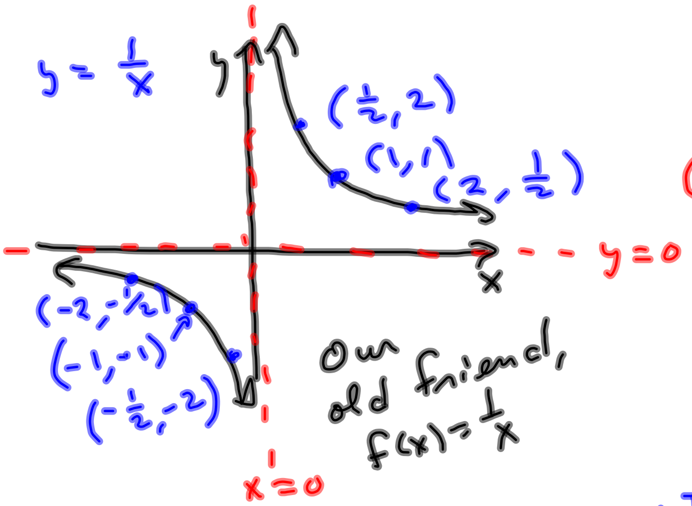
$x = \frac{1}{1000} \Longrightarrow f(x) = 1000$

$\frac{1}{\frac{1}{100}}$

$\frac{1}{\frac{1}{100}} = \frac{\frac{1}{1}}{\frac{1}{100}} = \frac{1}{1} \cdot \frac{100}{1}$

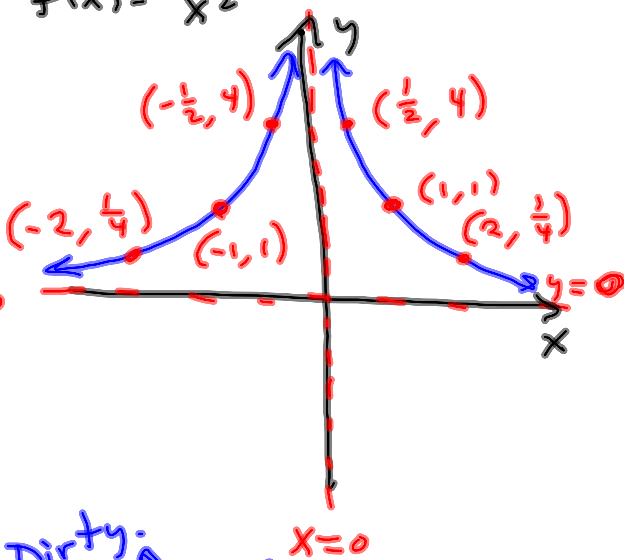
$1 \div \frac{1}{100} = 1 \cdot \frac{100}{1}$

8 $f(x) = \frac{1}{x^2}$



Our old friend, $f(x) = \frac{1}{x}$

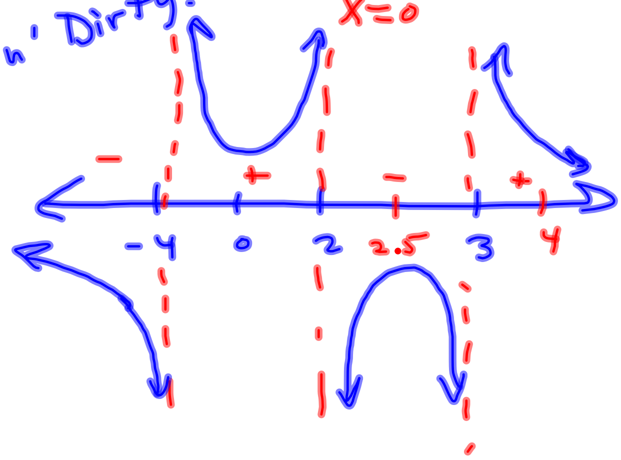
$f(x) = \frac{1}{x^2}$



Quick 'n' Dirty.

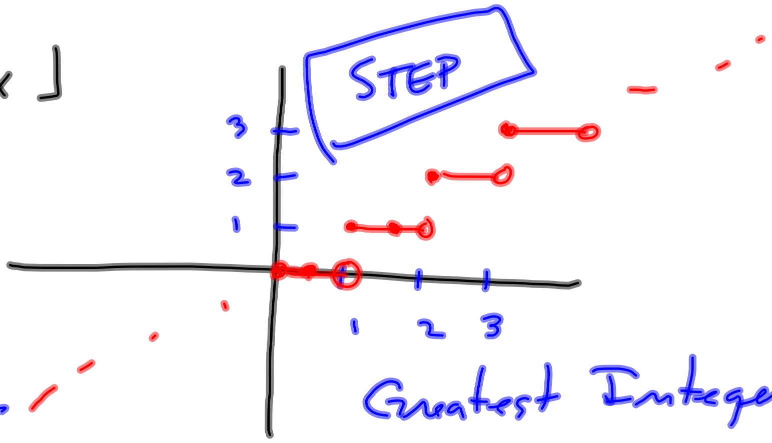
$$\frac{1}{(x-2)(x+4)(x-3)}$$

$(x-2)$	$(x+4)$	$(x-3)$
4	4	4
2.5	2.5	2.5
+	+	−
0	0	0
−	+	−



9) $f(x) = \lfloor x \rfloor$

x	y
.5	0
1	1
1.5	1
2	2



Just to see

I'm not big on this, but just in case you see it down the road.

Greatest Integer less than or equal to x.

§2.1 #s 1-8, 13, 14, 17-20, 23-28, 33-36

55, 56, 71-74, 83, 84, 87, 88, 91, 92, 95, 96

Practical for #s 33-36

"Does the equation define y as $f(x)$?"

If you can isolate y algebraically and get one expression involving x, then yes.

Yes, except for $|y|$, y^2, y^{2n}.

$3x+2y=7$

$2y=7-3x$

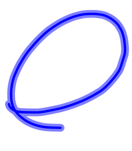
$y=\frac{7-3x}{2}$ one x in $\Longrightarrow$ one y out.

$x^2+y^2=25$ Not.

$y^2=25-x^2$

$\sqrt{y^2}=\sqrt{25-x^2}$

$|y|=\sqrt{25-x^2}$



$y=\pm\sqrt{25-x^2}$

$x=0$ corresponds to

$y=\pm 5 \rightsquigarrow (0,5)$ & $(0,-5)$

are pairs in the relation.

NOT A FUNCTION.

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$x^2+y^2=25 \longrightarrow y=\pm\sqrt{25-x^2}$

is a "special situation."

$x^2+y^2=25$ is a circle,

$\left\{\begin{array}{l} y=\sqrt{25-x^2} \text{ is its top half} \\ y=-\sqrt{25-x^2} \text{ is '' bottom half.} \end{array}\right.$

→ Graphing calculator can graph the two halves separately.

Book has $y=\sqrt{4-x^2}$ how to graph?

(from $x^2+y^2=4$)

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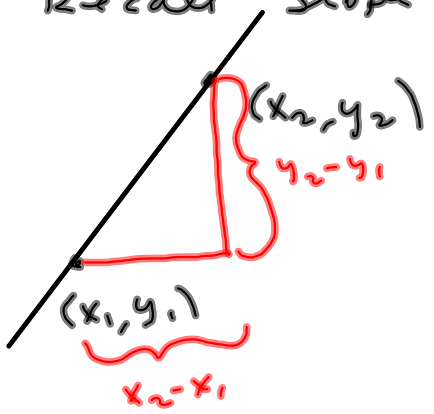
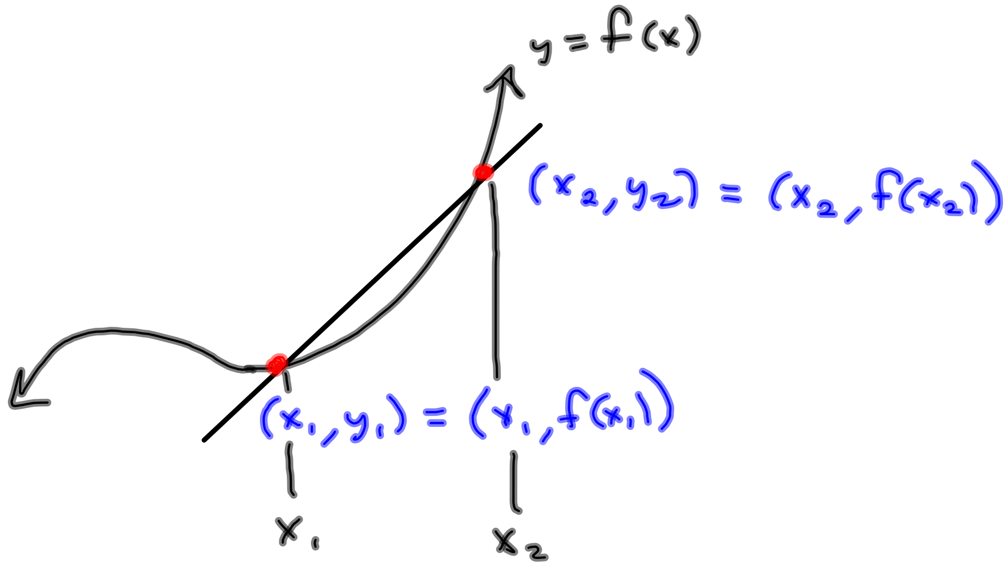
New from 2.1: Difference Quotient

Recall Slope between (x_1, y_1) & (x_2, y_2)



$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

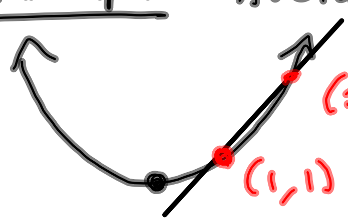
We generalize to slope of a CURVE, by findin average slope.



Average slope of $f(x)$ on $[x_1, x_2]$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

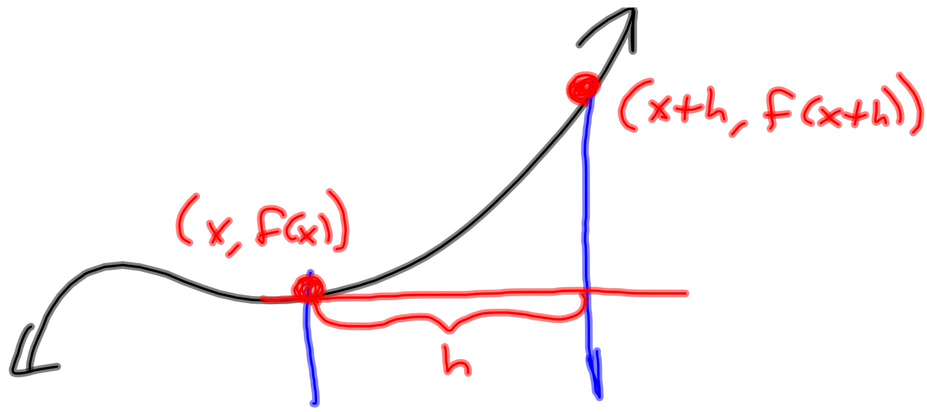
Example Average value of $f(x) = x^2$ on $[1, 2]$



$(2,4) = (x_2, f(x_2)) = (2, f(2))$

$(1,1) = (x_1, f(x_1)) = (1, f(1))$

$$m_{AVG} = \frac{f(2) - f(1)}{2 - 1} = \frac{4 - 1}{2 - 1} = \frac{3}{1} = 3$$



x_1 x_2

x $x+h$

$x_2 - x_1$ becomes

$x+h-x = h$

Standard Formulation of the

DIFFERENCE QUOTIENT

$$= m_{AVG} = \frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{f(x+h)-f(x)}{h}$$

MAT 201 ← $\xrightarrow{h \to 0}$ $f'(x) =$ slope @ one point!

Find (& simplify) the difference quotient for $f(x) = x^2$

$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$

want:

$$\frac{f(x+h) - f(x)}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \frac{2xh + h^2}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}} = \boxed{2x+h}$$

Trickeration

$f(x) = \sqrt{x}$

$(a - b)(a + b) = a^2 - b^2$

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$\frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{\sqrt{x+h} + \sqrt{x}}$$