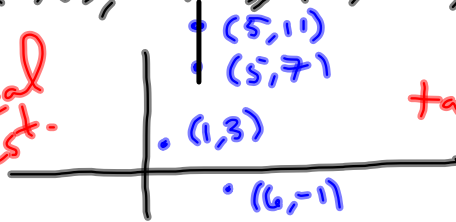


Function - A rule that assigns to each $x \in \text{Domain}$ to one $y \in \text{Range}$.

A relation R is a collection (set) of ordered pairs.

$$R = \{ (1, 3), (5, 7), (6, -1), (5, 11) \} \text{ is}$$

Not a
function.
Funks
vertical
line
test.



Graphical representation of example from Friday.

$$f = \{ (1, 3), (5, 7), (6, -1) \} \text{ is a function}$$

$$f(x) = 3x + 1 \Rightarrow$$

$$f(\text{☺}) = 3\text{☺} + 1$$

$$\begin{aligned} f(7x-11) &= 3(7x-11) + 1 \\ &= 21x - 33 + 1 \\ &= 21x - 32 \end{aligned}$$

$$f(r) = 3r + 1$$

$$f(x) = 2x^2 + 5x - 2$$

$$\begin{aligned} f(x+h) &= 2(x+h)^2 + 5(x+h) - 2 \\ &= 2(x^2 + 2xh + h^2) + 5x + 5h - 2 \\ &= 2x^2 + 4xh + 2h^2 + 5x + 5h - 2 \end{aligned}$$

$$(x+h)^2 = \underbrace{x^2 + 2xh + h^2}_{\substack{1x^2h^0 + 2x^1h^1 + 1x^0h^2}}$$

Pascal's Triangle.

$$\begin{array}{cccc} & & 1 & & \\ & & & 1 & \\ & 1 & & 2 & & 1 \\ & & 1 & & 3 & & 3 & & 1 \\ & & & 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

$$\begin{array}{r} 49 \\ \underline{7} \\ 6343 \\ \underline{-49} \\ 294 \end{array}$$

$$x^2 + 2xh + h^2$$

$$\frac{2h}{2} = h \rightarrow h^2$$

$$\begin{aligned} (x+h)^3 &= 1x^3h^0 + 3x^2h^1 + 3x^1h^2 + 1x^0h^3 \\ &= x^3 + 3x^2h + 3xh^2 + h^3 \end{aligned}$$

$$(2x+7)^3 = 1(2x)^3(7)^0 + 3(2x)^2(7)^1 + 3(2x)^1(7)^2 + (2x)^0(7)^3$$

$$\begin{aligned} &= 2^3x^3 \cdot 1 + 3(2^2x^2)(7) + 3 \cdot 2x \cdot 49 + 343 \\ &= 8x^3 + 84x^2 + 294x + 343 \end{aligned}$$

Examples of functions (families)

Func.

$$\textcircled{1} \quad g(x) = ax + b$$

linear function

$$\textcircled{2} \quad g(x) = ax^2 + bx + c$$

quadratic function

$$\textcircled{3} \quad g(x) = ax^3 + bx^2 + cx + d$$

cubic function

$$\textcircled{4} \quad g(x) = a\sqrt{bx+c} + d$$

square root function

$$\textcircled{5} \quad g(x) = a\sqrt[3]{bx+c} + d$$

$$\textcircled{6} \quad g(x) = a|bx+c| + d$$

Basic Func.

$$f(x) = x$$

identity function

$$f(x) = x^2$$

squaring function
square

$$f(x) = x^3$$

cube function
cubing

$$f(x) = \sqrt{x}$$

square root
function

$$f(x) = \sqrt[3]{x}$$

cube root
function

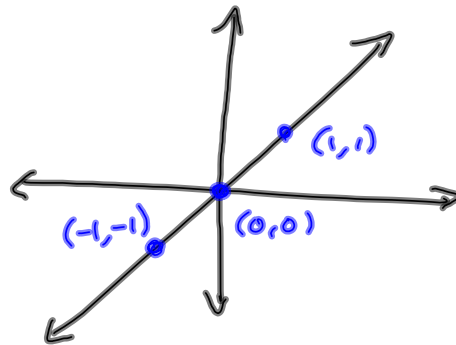
$$f(x) = |x|$$

absolute value.

① $f(x) = x$: identity

$$y = x$$

x	y
-1	-1
0	0
1	1



$f(x-3)$

move right 3 units.

Delay by 3 units.

Add 3 to every x-coord!

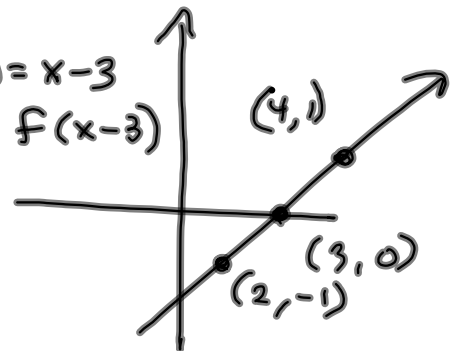
$$y = x - 3$$

$$f(x) = x$$

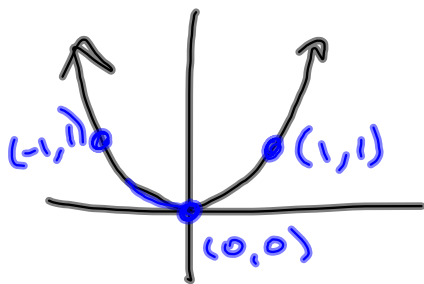
$$f(x-3) = x - 3$$

$$g(x) = x - 3$$

$$= f(x-3)$$

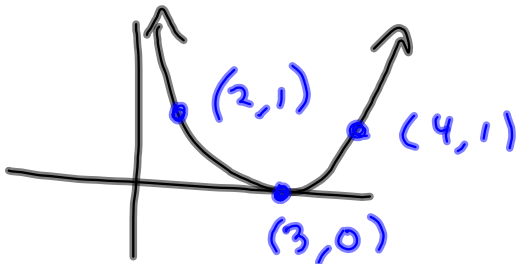


② Same deal with $f(x) = x^2$



x	y = x ²
-1	1
0	0
1	1

$$f(x-3) = (x-3)^2 = g(x)$$



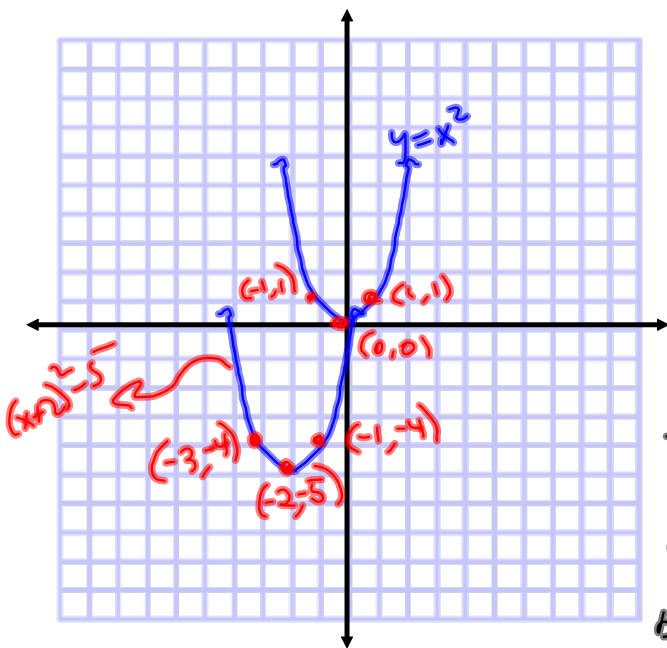
$f(x+3)$ left 3

$f(x) - 3$ down 3

$f(x) + 3$ up 3

} Vertical
(rigid)
shifts.

Rigid transformations



$$f(x) = x^2$$

$$g(x) = (x+2)^2 - 5$$

$$= f(x+2) - 5$$

Left 2

Down 5

graph $g(x) = x^2 + 4x - 1$
by shifting / stretching
 $f(x) = x^2$

Didn't
involve a
stretch.

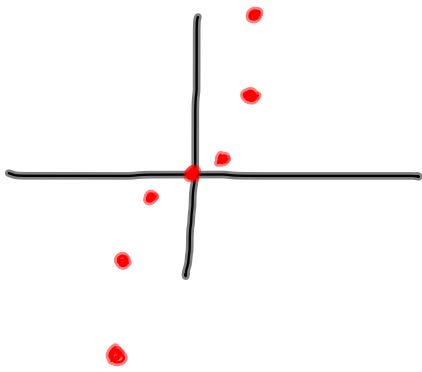
$$g(x) = x^2 + 4x - 1$$

$$= x^2 + 4x + 2^2 - 4 - 1$$

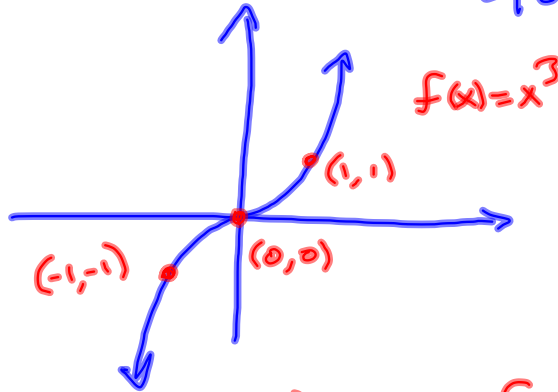
$$\frac{4}{2} = 2 \rightarrow 2^2 = 4$$

$$= (x+2)^2 - 5 \quad \text{See graph.}$$

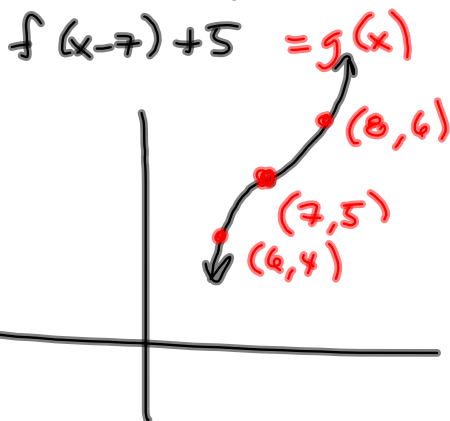
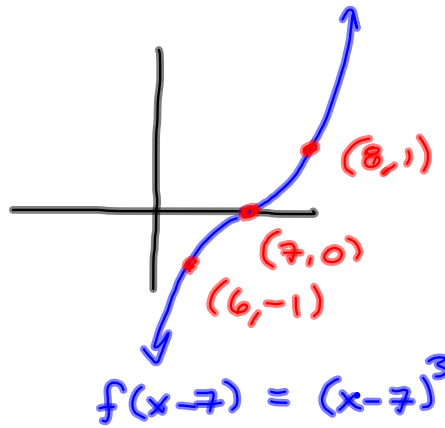
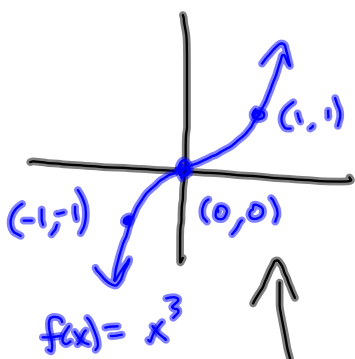
③ Cubic $f(x) = x^3$



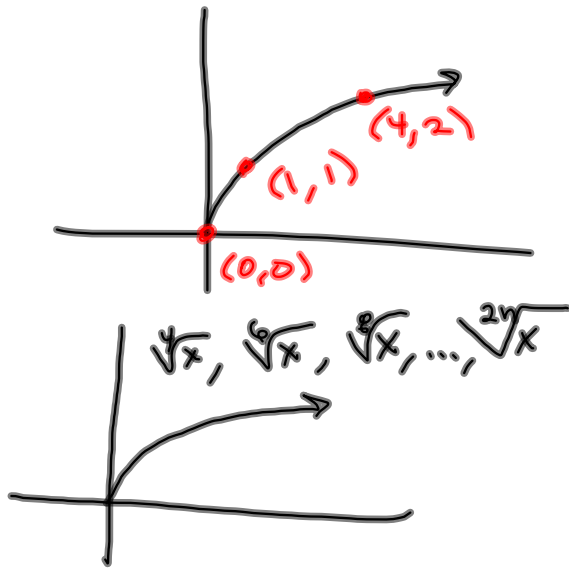
x	y
-2	$(-2)^3 = -8$
-1	-1
0	0
1	1
2	8



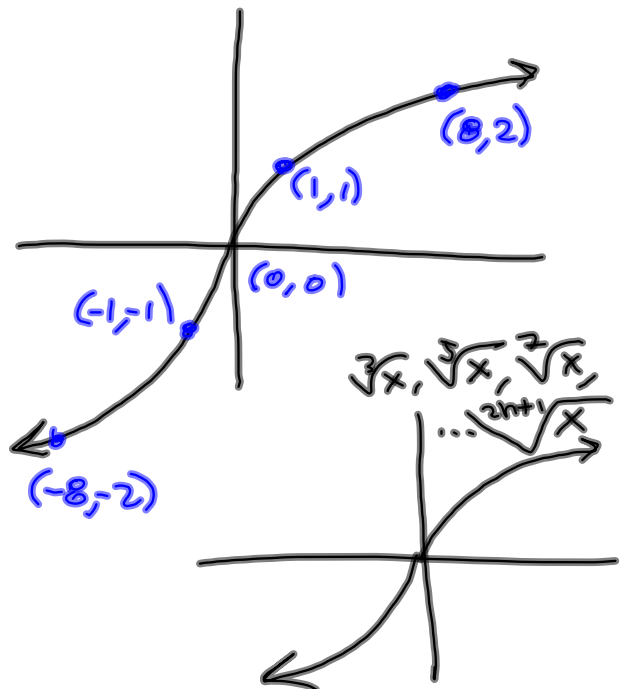
$g(x) = (x-7)^3 + 5 = f(x-7) + 5$ for $f(x) = x^3$
 ↑ Right 7 ↑ up 5



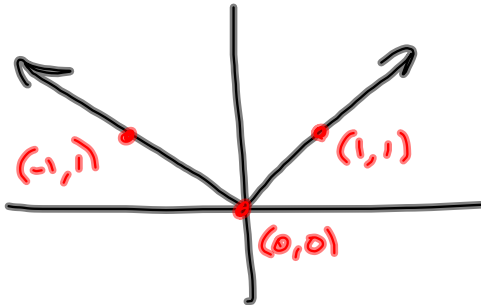
④ $f(x) = \sqrt{x}$



⑤ $f(x) = \sqrt[3]{x}$



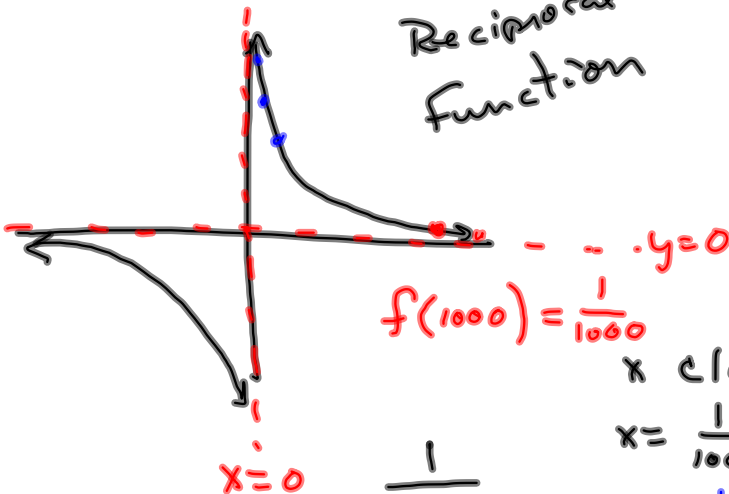
⑥ $f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$ piecewise def'n.



⑦ $f(x) = \frac{1}{x}$
Reciprocal Function

x	y
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
0	∞
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$

$\frac{1}{0}$



$f(1000) = \frac{1}{1000}$

$\frac{1}{\frac{1}{100}}$

x close to zero:

$x = \frac{1}{100} \Rightarrow f(x) = 100$

$x = \frac{1}{1000} \Rightarrow f(x) = 1000$

$\frac{1}{\frac{1}{100}} = \frac{\frac{1}{1}}{\frac{1}{100}} = \frac{1}{1} \cdot \frac{100}{1}$

$1 \div \frac{1}{100} = 1 \cdot \frac{100}{1}$

