Square Root Property $\Longleftrightarrow$ Completing the Square.

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

$(x+y)^2 = 1x^2 + 2xy + y^2$

$(3x-5)^2 = (3x)^2 + 2(3x)(-5) + (-5)^2$

$= 9x^2 - 30x + 25$

$(4x+2y)^2 = (4x)^2 + 2(4x)(2y) + (2y)^2$

$= 16x^2 + 16xy + 4y^2$

$(2y)^2 = 2^2 y^2$

$A^2 = B \Longrightarrow$

$\sqrt{A^2} = \sqrt{B} \Longrightarrow$

$|A| = \sqrt{B} \Longrightarrow$

$A = \pm\sqrt{B}$

$x^2 - 2 = 0$

$(x-\sqrt{2})(x+\sqrt{2})=0$, using $2 = \sqrt{2}^2$ & $a^2-b^2 = (a-b)(a+b)$

No x-term:

$x^2 = 2$

$\vdots$ Square Root Property

$x = \pm\sqrt{2}$

$(2x-3)^2 + 5 = 2$

$(2x-3)^2 = -3$

$\sqrt{-3} = \sqrt{(-1)(3)}$
$= \sqrt{-1}\sqrt{3} = i\sqrt{3}$

$2x-3 = \pm\sqrt{-3} = \pm i\sqrt{3}$

$2x = 3 \pm i\sqrt{3}$

$x = \dfrac{3 \pm i\sqrt{3}}{2}$

when you <u>can</u> use it, square root property is slick

Completing the Square to solve $ax^2+bx+c=0$

$3x^2-2x+7=0$

$x^2-\frac{2}{3}x+\frac{7}{3}=0$

$x^2-\frac{2}{3}x \qquad = -\frac{7}{3}$

$\frac{\frac{2}{3}}{2}=\frac{2}{3}\cdot\frac{1}{2}=\frac{1}{3} \rightarrow \left(\frac{1}{3}\right)^2=\frac{1}{9}$

$x^2-\frac{2}{3}x+\left(\frac{1}{3}\right)^2=-\frac{7}{3}+\frac{1}{9}$

$-\frac{7}{3}\cdot\frac{3}{3}+\frac{1}{3\cdot3}=\frac{-20}{9}$

$\left(x-\frac{1}{3}\right)^2=-\frac{20}{9}$

$x-\frac{1}{3}=\pm\sqrt{-\frac{20}{9}}=\pm i\sqrt{\frac{20}{9}}$

$x-\frac{1}{3}=\pm\frac{2\sqrt{5}}{3}i$

$x=\frac{1}{3}\pm\frac{2\sqrt{5}}{3}i=\frac{1\pm2\sqrt{5}i}{3}$

$\sqrt{\frac{20}{9}}=\sqrt{\frac{2\cdot2\cdot5}{3\cdot3}}=\frac{2}{3}\sqrt{5}=\frac{2\sqrt{5}}{3}$

Completing the square to re-write & graph

$f(x)=ax^2+bx+c$

$f(x)=3x^2-2x+7$

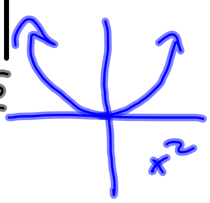
$=3\left(x^2-\frac{2}{3}x\right)+7$

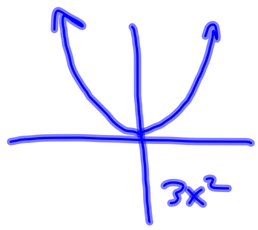
$=3\left(x^2-\frac{2}{3}x+\left(\frac{1}{3}\right)^2\right)+7-\frac{1}{3}$

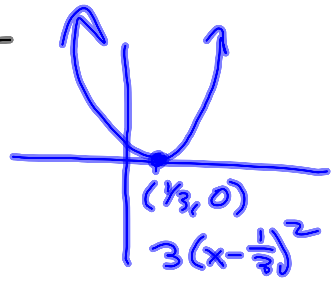
I added $3\left(\frac{1}{9}\right)=\frac{1}{3}$

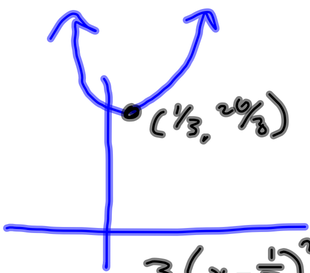
$=3\left(x-\frac{1}{3}\right)^2+\frac{20}{3}$

3 times taller Right $\frac{1}{3}$ up $\frac{20}{3}$









$3\left(x-\frac{1}{3}\right)^2+\frac{20}{3}=f(x)$

$ax^2+bx+c=0$ Assume $a>0$

$x^2+\frac{b}{a}x+\frac{c}{a}=0$

$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=-\frac{c}{a}+\frac{b^2}{4a^2}$

$\frac{\frac{b}{a}}{2}=\frac{b}{2a} \rightsquigarrow \left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}$

$\left(x+\frac{b}{2a}\right)^2=-\frac{c}{a}+\frac{b^2}{4a^2}$

$\frac{b^2}{4a^2}-\frac{c}{a}\cdot\frac{4a}{4a}$

$=\frac{b^2-4ac}{4a^2}$

$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$

$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}=\pm\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}$

$=\pm\frac{\sqrt{b^2-4ac}}{2|a|}$

$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

$=\pm\frac{\sqrt{b^2-4ac}}{2a}$ if $a>0$, & it is!

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

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Cheat:

$$f(x) = ax^2 + bx + c$$

$$= \boxed{a\left(x+\frac{b}{2a}\right)^2 + f\left(-\frac{b}{2a}\right)}$$

$$= 3\left(x-\frac{1}{3}\right)^2 + \frac{20}{3}$$

$$f(x) = 3x^2 - 2x + 7$$

$$a=3,\ b=-2,\ c=7$$

$$\frac{b}{2a} = \frac{-2}{2\cdot 3} = -\frac{1}{3}$$

$$f\left(-\frac{b}{2a}\right) = 3\left(\frac{1}{3}\right)^2 - 2\left(+\frac{1}{3}\right) + 7$$

$$= \frac{1}{3} - \frac{2}{3} + 7 = \frac{20}{3}$$

$$= 3\left(x-\frac{1}{3}\right)^2 + \frac{20}{3}$$

Vertex: $\left(\frac{1}{3}, \frac{20}{3}\right)$

$$= \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

$$f(x) = ax^2 + bx + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 + f\left(-\frac{b}{2a}\right)$$

Applied to Solving Equations

Solve $3x^2 - 2x + 7 = 0$ by completing the square.

$$3\left(x - \frac{1}{3}\right)^2 + \frac{20}{3} = 0$$

$$3\left(x - \frac{1}{3}\right)^2 = -\frac{20}{3}$$

$$\left(x - \frac{1}{3}\right)^2 = -\frac{20}{9}, \text{ etc.}$$

This would trick me.

SOLVE by factoring: $15x^2 - 8x - 12 = 0$

Magic #: $(15)(12) = -180$

$-8 = -9 + 1$ $(9)(1) = -9$

$= -10 + 2$ -20

$= -11 + 3$ -33

$= -12 + 4$ -48

$= -13 + 5$ -65

$= -23 + 15$ -345

$= -15 + 7$ -105

$= -18 + 10$ -180

$15x^2 - 18x + 10x - 12 =$

$3x(5x-6) + 2(5x-6) =$

$(5x-6)(3x+2) = 0$

$5x - 6 = 0$ or $3x + 2 = 0$

$5x = 6$ $3x = -2$

$x = \frac{6}{5}$ OR $x = -\frac{2}{3}$

other methods have you play with factors of -180 that add up to -8. This method starts with the -8 & just walks you to a product of -180.

$15x^2 - 8x - 12 = 0$

When in doubt, use quadratic formula and

Quadratic Formula FACTOR THEOREM

to find $\frac{6}{5}, -\frac{2}{3}$

Then write $15\left(x - \frac{6}{5}\right)\left(x + \frac{2}{3}\right)$

$= 3(5x-6)\left(x+\frac{2}{3}\right)$

$= \boxed{(5x-6)(3x+2)}$

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