PRACTICE TEST #3

1. State whether the function is a polynomial or not. If not, give a reason why. (probably won't see this on test, but still good to know!)

   a. $f(x) = \sqrt{x^2 + 5} - 14x^3$

   b. $f(x) = x^3 + 3x^2 + \frac{1}{x}$

   c. $f(x) = \frac{3x^3 + 9(x-3)^2}{3}$

2. Form a polynomial with real coefficients that has the given zeros and degree.

   a. Zeros: 3, multiplicity 2; -2, multiplicity 3; 6, multiplicity 1. Degree 6

   b. Zeros: 2, multiplicity 1; 5, multiplicity 2; $3 + 2i$, multiplicity 1. Degree 5

3. Expand $\big(x - (2 + 5i)\big)\big(x - (2 - 5i)\big)$

4. Let $f(x) = 3(x-2)^3(x+4)(x-5)^2$

a. List each real zero and its multiplicity. Determine whether the graph of $f(x)$ touches or crosses the x-axis at each x-intercept.

b. Determine the power function that $f(x)$ resembles as $x \to \pm\infty$. This is the End Behavior part of the question. (i.e. determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$)

d. Use the information you reported to obtain a rough graph of $f(x)$.

5. Find the asymptotes (i.e. vertical, and/or horizontal and/or oblique). Reminder: you find the vertical asymptotes by finding where the denominator equals zero. For Part ii, you will need to use long division to find the slant asymptote.

i) $R(x) = \frac{120x^4+5594x^2-0.009x+2}{-12x^4+x^3}$

ii) $G(x) = \frac{x^3-8}{x^2-5x+6}$

6. Solve the inequalities (Hint: use a sign patter.)

a. $(x-2)(x+3)^2(x-7)^3 \geq 0$

b. $\frac{x-3}{(x+5)^2(x-7)^3} \geq 0$

7. Graph the function $R(x) = \frac{x^3-3x^2-13x+15}{x^3-5x^2-14x+16} = \frac{(x-1)(x+3)(x-5)}{(x+2)(x-1)(3x-8)}$. Key features are asymptotes, holes (if any) and intercepts.

8. Use Descarte's Rule of Signs and the Rational Zeros Theorem to find all the real zeros of $f(x) = x^4 - 6x^3 + 7x^2 - 6x - 20$. Use the *real* zeros to factor *f* over the real numbers. This is likely to involve an irreducible quadratic factor.

9. Based on your work in #8 above, find *all* the (real and nonreal) zeros of $f(x) = x^4 - 6x^3 + 7x^2 - 6x - 20$. Use *all* the zeros to write $f(x)$ as the product of *linear* factors.

10. Sketch the graph of $R(x) = \frac{6x^3-7x^2-14x+15}{2x^2-5x+3}$. State the domain, asymptotes, holes, and intercepts. Show them clearly labeled on your graph. (Hint: factor the denominator, then use the zeros of the denominator to check for zeros of the numerator using synthetic division.)

11. What is the domain of $\sqrt{\frac{x-2}{(x+3)^2(x-7)^3}}$?

12. List each real zero and its multiplicity. Determine whether the graph of $f(x)$ touches or crosses the x-axis at each x-intercept.

$$f(x) = 4x(x^2 - 4)(x^3 + 1)^2$$

13. Solve each of the following quadratic inequalities. Express your answer interval notation.

a. $x^2 < x + 12$

b. $x^2 - x > 11$

14. Solve the following absolute value inequalities and equations. Draw a quick sketch if it is helpful.

a. $|2x - 2| = 8$

b. $|2 - 2x| < -8$

c. $|2x - 2| < 8$

d. $|2 - 2x| > -8$

15. Use the Remainder Theorem to find $P(3)$ if $P(x) = 2x^3 + 3x^4 - 5x^2 + 4x - 6$

16. Use the Intermediate Value Theorem to show that the polynomial function has a zero in the given interval. (Use synthetic division if you want to make Steve really happy!)

$f(x) = 2x^3 + 6x^2 - 8x + 2; \quad [-5, -4]$

17. Find all solutions (both real and non-real) to each equation. Check your answers.

i) $k(x) = \sqrt{3x - 5 = 4}$

ii) $G(x) = x^4 - 4x^2 - 7 = -2$

iii) $(u^2 + 2u) - 2(u^2 + 2u) = 3$

iv) h(x) = $\sqrt{x + 40} - \sqrt{x} = 4$

18. Put the following function into the form $a(x-h)^2+k$ and graph it. State the domain and range of the function.

$y = x^2 - 6x + 11$