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This Test is Due Friday, November 16th

1. In each of the following, form a polynomial with real coefficients that has the given zeros and degree. Please do not expand the polynomial.
a. (5 pts) Zeros: - 4, multiplicity 2; 2, multiplicity 3. Degree 5.
b. (5 pts) Zeros: 2, multiplicity 1; 5, multiplicity 2; 7 - 8i, multiplicity 1. Degree 5.
2. (5 pts) Expand $(x-(3+6 i))(x-(3-6 i))$
3. (5 pts) Use synthetic division to find $P(2)$ if $P(x)=x^{4}-5 x^{3}+11 x^{2}-12 x+13$.
4. (5 pts) Divide $f(x)=2 x^{4}-3 x^{3}+x-3$ by $f(x)=x^{2}-1$
5. Let $f(x)=2(x-1)^{2}(x+4)(x-5)^{3}$.
a. (5 pts) List each real zero and its multiplicity. Determine whether the graph of $f(x)$ touches or crosses the $x$-axis at each $x$-intercept.
b. (5 pts) Find the following limits (This relates to end behavior):
i. $\lim _{x \rightarrow \infty} f(x)$
ii. $\lim _{x \rightarrow-\infty} f(x)$
c. (5 pts) Use your work, above, to help you sketch the graph of $f(x)$, showing all intercepts (including the $y$-intercept).
6. Solve the inequalities (Hint: You already laid the foundations for both of these in the previous problem.).
a. $\left(10\right.$ pts) $2(x-1)^{2}(x+4)(x-5)^{3} \geq 0$
b. (5 pts) $\frac{(x+4)(x-1)^{2}}{(x-5)^{3}} \geq 0$ (Hint: This one differs only slightly from the previous one.)
7. ( 10 pts) Find all the zeros of $f(x)=x^{4}-5 x^{3}+15 x^{2}-5 x-26$. Write $f$ in factored form, using the zeros you find.
8. (10 pts) Graph the function $R(x)=\frac{2 x^{3}-3 x^{2}-2 x+3}{x^{3}-4 x^{2}+x+6}=\frac{(x-1)(2 x-3)(x+1)}{(x+1)(x-2)(x-3)}$. Key features are asymptotes, holes (if any) and intercepts. I was kind enough to factor it for you.
9. (10 pts) Sketch the graph of $f(x)=\frac{x^{3}-x+2 x^{2}-2}{x^{2}-2 x-3}=\frac{(x+2)\left(x^{2}-1\right)}{(x+1)(x-3)}$. Show all intercepts, asymptotes and holes it has, if any.
