121-G12
Fall, 2012
Practice (See Syllabus)

Test 1

Name KEY

1. Solve each equation. Identify each equation as an identity, inconsistent, or conditional equation.

a. $\frac{1}{w-1}-\frac{1}{2w-2}=\frac{1}{2w-2}$ $\quad LCD = 2(w-1)$

$2(w-1)$

$\frac{1}{w-1}\cdot\frac{2}{2}-\frac{1}{2(w-1)}=\frac{1}{2(w-1)}$

$2-1=1$

$1=1$

IDENTITY

Sol'n Set: $\{w \mid w \neq 1\}$

b. $\frac{z-3}{z+2}=-\frac{5}{3}$ $\quad LCD = 3(z+2)$

$\frac{3}{3}\cdot\frac{z-3}{z+2}=-\frac{5}{3}\cdot\frac{z+2}{z+2}$

$3(z-3)=-5(z+2)$

$3z-9=-5z-10$

$8z=-1$

$z=-\frac{1}{8}$

$z \in \{-\frac{1}{8}\}$

2. Solve the absolute value equations

a. $|x-4|=8$

$x-4=8$ OR $x-4=-8$

$\{x \mid x=12$ OR $x=-4\}$

$=\{-4, 12\}$

b. $2|x+5|-10=0$

$2|x+5|=10$

$|x+5|=5$

$x+5=5$ OR $x+5=-5$

$\{x \mid x=0$ OR $x=-10\}$

$=\{-10, 0\}$

c. $|x+8|=-3$

3. If I buy a 2012 Tacoma for \$34,000 and that price includes 7% sales tax, then how much does the truck cost *before* sales tax?

x = price before tax (\$)

$x + .07x = 34000$

$1.07x = 34000$

$x = \frac{34000}{1.07} \approx \$31,775.70$

4. Johnny splits a \$12,000 investment into two smaller investments. The higher-risk account has a rate of return of 7% and the lower-risk account has a rate of return of 5%. If Johnny earns \$740 in interest after one year, how much did he invest in each account?

Let x = amt invested @ 7% (\$)

y = " " " 5% (\$)

$x + y = 12000$

$.07x + .05y = 740$

$.07x + .05(12000 - x) = 740$

$.07x + 600 - .05x = 740$

$.02x = 140$

$x = \frac{140}{.02} = 7000 = x \Rightarrow$

$y = \$5000$

5. Jim can stack 500 hay bales in 3 hours. It takes Jenny 4 hours to stack 500 hay bales. How long does it take the two of them to stack 500 hay bales if they work together?

t = time it takes them working together (hrs)

$\frac{1}{3}t + \frac{1}{4}t = 1$

$\frac{4t + 3t}{12} = \frac{12}{12}$

$7t = 12 \Rightarrow t = \frac{12}{7} \approx 1.714285714$

6. Suppose Jenny starts stacking hay bales at 7 a.m. and Jim doesn't join her until 8 a.m. To the nearest *minute*, what time will they finish?

t = time Jenny spends on the job. (hrs)

Then

$\frac{1}{4}t + \frac{1}{3}(t-1) = 1$

$\frac{3t + 4(t-1)}{12} = \frac{12}{12}$

$3t + 4t - 4 = 12$

$7t = 16$

$t = \frac{16}{7} \approx 2.285714286$

$(.285714286\ \text{hr})\left(\frac{60\ \text{min}}{\text{hr}}\right) \approx 17.143$

7am + 2hrs + 17 min $\approx$ 9:17am

7. Find the distance between and the midpoint of the two points $P(2, 5)$ and $Q(-3, 9)$.

$d(P,Q) = \sqrt{(2+3)^2 + (5-9)^2}$ $\quad mid(P,Q) = \left(\frac{2-3}{2}, \frac{5+9}{2}\right)$

$= \sqrt{5^2 + 4^2}$ $\quad = \left(-\frac{1}{2}, \frac{14}{2}\right)$

$= \sqrt{25+16}$ $\quad = \left(-\frac{1}{2}, 7\right)$

$= \sqrt{41} = d(P,Q) \times 3.741657387$

Distance = $\sqrt{41}$ **Midpoint =** $\left(-\frac{1}{2}, 7\right)$

8. Determine the center and radius of the circle and sketch its graph:

$$x^2 + y^2 + 6x - 14y + 58 = 25$$

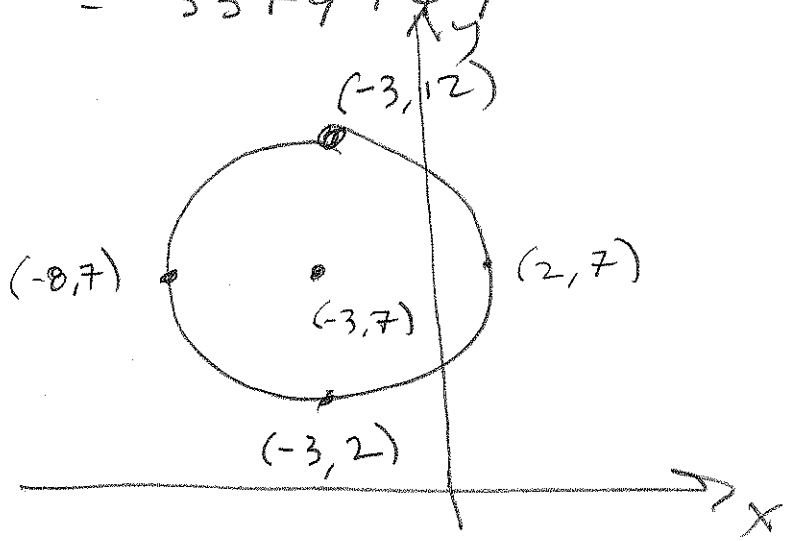
$x^2+6x \quad + y^2-14y \quad = -33$

$x^2+6x+3^2+y^2-14y+7^2 = -33+9+49$

$(x+3)^2+(y-7)^2 = 25$

$(h,k) = (-3,7)$

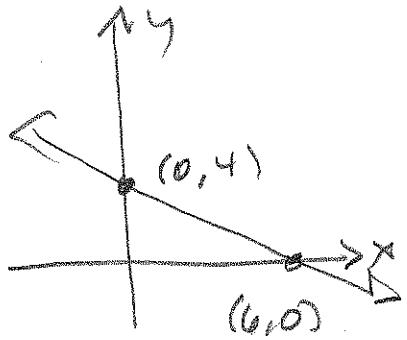
$r = 5$



9. Graph each equation. Show any x- or y-intercepts.

a. $2x+3y=12$

x	y
0	4
6	0



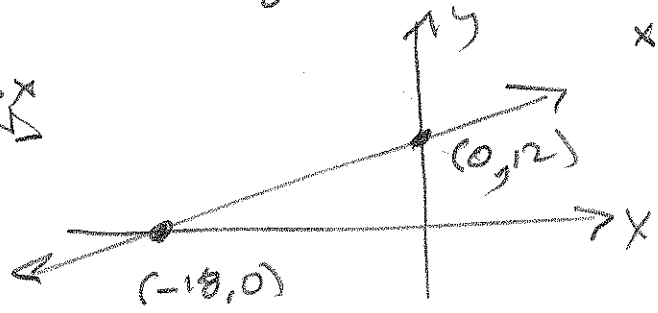
b. $y = \frac{2}{3}x + 12$ SET 0

x	y
0	12
-18	0

$\frac{2}{3}x = -12$

$2x = -36$

$x = -18$



10. Write an equation of the line through the points $P(2, 5)$ and $Q(-3, 9)$. Express the equation in all three forms:

i. Point-Slope
ii. Slope-Intercept
iii. Standard

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-5}{-3-2} = \frac{4}{-5}$

(i) $y = -\frac{4}{5}(x-2) + 5$

$= -\frac{4}{5}x + \frac{8}{5} + \frac{25}{5}$

(ii) $y = -\frac{4}{5}x + \frac{33}{5}$

$5y = -4x + 33$

(iii) $4x + 5y = 33$

11. Write an equation in point-slope form of the line through $P(3, -7)$ that is...

a. ... parallel to the line $y = \frac{3}{\pi}x + \frac{11}{97}$ $\quad y = \frac{3}{\pi}(x-3) - 7$

b. ... perpendicular to the line $y = \frac{3}{\pi}x + \frac{11}{97}$ $\quad y = -\frac{\pi}{3}(x-3) - 7$

12. Solve $x^2 - 11x - 42 = 0$ in two ways:

a. Completing the square

$x^2 - 11x = 42$

$x^2 - 11x + \left(\frac{11}{2}\right)^2 = 42 + \frac{121}{4}$

$\left(x - \frac{11}{2}\right)^2 = \frac{289}{4}$

$x - \frac{11}{2} = \pm\sqrt{\frac{289}{4}} = \pm\frac{17}{2}$

$x = \frac{11 \pm 17}{2}$ $\quad \frac{28}{2} = 14$, $\frac{-6}{2} = -3$

$x \in \{-3, 14\}$

b. Quadratic formula

$a = 1, b = -11, c = -42$

$b^2 - 4ac = (-11)^2 - 4(1)(-42)$

$= 121 + 168 = 289$

$x = \frac{11 \pm \sqrt{289}}{2(1)} = \frac{11 \pm 17}{2} \Rightarrow x \in \{-3, 14\}$

13. Solve the inequalities. Give your answers in two forms:
    i. Set-Builder Form
    ii. Interval Notation

a. $3x-2>4$ and $17-2x\geq -5$

$3x>6$ & $-2x\geq -22$

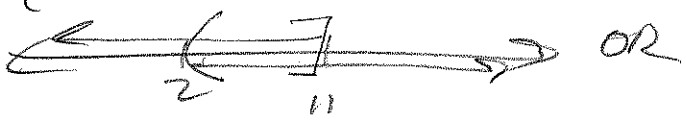
$\{x \mid x>2$ & $x\leq 11\}$

$= (2, 11]$

b. $3x-2>4$ or $17-2x\geq -5$

$3x>6$ OR $-2x\geq -22$

$\{x \mid x>2$ OR $x\leq 11\}$



$= (-\infty, \infty) = \mathbb{R}$

c. $3x+10<5$ or $2x-13>27$

$3x<-5$ OR $2x>40$

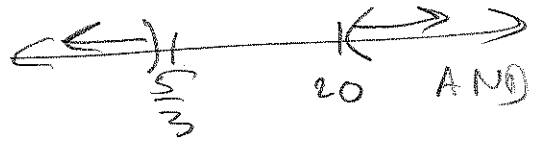
$\{x \mid x<-\frac{5}{3}$ OR $x>20\}$

$= (-\infty, -\frac{5}{3}) \cup (2, \infty)$

d. $3x+10<5$ and $2x-13>27$

$3x<5$ & $2x>40$

$\{x \mid x<\frac{5}{3}$ & $x>20\}$



$= \varnothing$ No Soln

e. $|3x-2|\geq 4$

$3x-2\geq 4$ OR $3x-2\leq -4$

$3x\geq 6$ OR $3x\leq -2$

$\{x \mid x\geq 2$ OR $x\leq -\frac{2}{3}\}$

$= (-\infty, -\frac{2}{3}] \cup [2, \infty)$

f. $|3x-2|\leq 4$

$3x-2\leq 4$ & $3x-2\geq -4$

$3x\leq 6$ & $3x\geq -2$

$\{x \mid x\leq 2$ & $x\geq -\frac{2}{3}\}$

$= \boxed{[-\frac{2}{3}, 2)}$

g. $|3x-2|\leq -4$ $\varnothing$

h. $|3x-2|\geq -4$ $\mathbb{R}$