

This is our final learning opportunity together, and I'm hoping to take full advantage. Read the questions carefully. Sometimes, you can earn points on a problem by *knowing* that you did it wrong and *explaining* how you know and what you're *trying* to accomplish, and *how* you're going about it.

1. Solve the equation $x^2 - 2x - 15 = 0$ in three different ways:

part a (10 pts) Factoring

$$(x-5)(x+3) = 0$$
$$x \in \{-3, 5\}$$

part b (15 pts) Completing the square

$$x^2 - 2x = 15$$
$$x^2 - 2x + 1 = 16$$
$$(x-1)^2 = 16$$
$$x-1 = \pm 4$$
$$x = 1 \pm 4$$

↖ 5 ↘ -3

$$x \in \{-3, 5\}$$

part c (15 pts) Quadratic formula

$$a=1, b=-2, c=-15$$
$$b^2 - 4ac = (-2)^2 - 4(1)(-15)$$
$$= 4 + 60 = 64$$
$$x = \frac{-(-2) \pm \sqrt{64}}{2(1)} = \frac{2 \pm 8}{2}$$

↖ $\frac{10}{2} = 5$
↘ $-\frac{6}{2} = -3$

$$x \in \{-3, 5\}$$

2. Solve the absolute value inequality. Give your answer in set-builder *and* interval notation.

part a (10 pts) $|7x+2| \geq 4$

part b (10 pts) $|2x-7| < 4$

$$7x+2 \geq 4 \quad \text{OR} \quad 7x+2 \leq -4$$

$$7x \geq 2 \quad \quad \quad 7x \leq -6$$

$$\left\{ x \mid x \geq \frac{2}{7} \quad \text{OR} \quad x \leq -\frac{6}{7} \right\}$$

$$(-\infty, -\frac{6}{7}] \cup [\frac{2}{7}, \infty)$$

$$-4 < 2x-7 < 4$$

$$3 < 2x < 11$$

$$\left\{ x \mid \frac{3}{2} < x < \frac{11}{2} \right\}$$

$$x \in (\frac{3}{2}, \frac{11}{2})$$

3. Let $f(x) = \sqrt{x-14}$ and $g(x) = x^2 - 3x - 14$

part a (15 pts) What's the domain of $f(x)$? Give the answer in set-builder and interval notation.

$$x-14 \geq 0$$

$$\left\{ x \mid x \geq 14 \right\} = [14, \infty)$$

part b (15 pts) Determine $(f \circ g)(x)$. Simplify your answer.

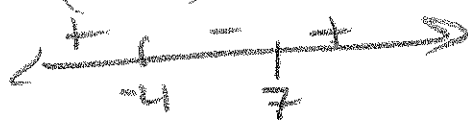
$$\sqrt{x^2 - 3x - 14 - 14} = \sqrt{x^2 - 3x - 28}$$

part c (5 pts) What's the domain of $(f \circ g)(x)$? Give your answer in set-builder and interval notation.

NEED $x^2 - 3x - 14 \geq 14 \Rightarrow$

$$x^2 - 3x - 28 \geq 0$$

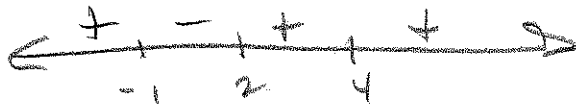
$$(x-7)(x+4) \geq 0$$



$$\left\{ x \mid x \leq -4 \quad \text{OR} \quad x \geq 7 \right\}$$

$$(-\infty, -4] \cup [7, \infty)$$

4. (15 pts) Solve $(x-2)^3(x+1)(x-4)^2 > 0$. Give the solution set in interval notation.



$$x \in (-\infty, -1) \cup (2, 4) \cup (4, \infty)$$

5. (10 pts) What is the domain of $h(x) = \sqrt{(x-2)^3(x+1)(x-4)^2}$?

$$x \in (-\infty, -1] \cup [2, 4] \cup [4, \infty)$$

$$= (-\infty, -1] \cup [2, \infty)$$

6. (10 pts) What is the domain of $\sqrt{\frac{(x-2)^3(x+1)}{(x-4)^2}}$? The hard part's done...

$x \neq 4$, but otherwise same as #5

$$x \in (-\infty, -1] \cup [2, 4) \cup (4, \infty)$$

7. (10 pts) Use synthetic division to find $f(3)$ for $f(x) = x^4 - 5x^3 - 3x^2 + 43x - 60$

$$\begin{array}{r|rrrrr} 3 & 1 & -5 & -3 & 43 & -60 \\ & & 3 & -6 & -27 & 48 \\ \hline & 1 & -2 & -9 & 16 & -12 = f(3) \end{array}$$

8. (10 pts) Determine a, r and n for the finite geometric sequence $2, \frac{4}{5}, \frac{8}{25}, \dots, \frac{256}{78125}$

Use a, r , and n to determine the sum by the formula $\sum_{k=1}^n a \cdot r^{k-1} = a \left(\frac{1-r^n}{1-r} \right)$. A

fractional answer is better, but I'll give you most of the points if you provide a decimal answer that is accurate to 4 decimal places.

$$\frac{256}{78125} = \frac{2^8}{5^7} = 2 \cdot \frac{2^7}{5^7}$$

$$2 \cdot \frac{2^7}{5^7} = 2 \cdot \frac{2^{n-1}}{5^{n-1}} \Rightarrow$$

$$n=8, a=2, r=\frac{2}{5}$$

$$\Rightarrow S = 2 \left(\frac{1 - \left(\frac{2}{5}\right)^8}{1 - \frac{2}{5}} \right)$$

$$\begin{array}{r} 5 \overline{)78125} \\ \underline{5} \\ 28125 \\ \underline{25} \\ 3125 \\ \underline{25} \\ 625 \\ \underline{5} \\ 125 \\ \underline{5} \\ 25 \\ \underline{5} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \overline{)256} \\ \underline{4} \\ 128 \\ \underline{256} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \overline{)128} \\ \underline{4} \\ 64 \\ \underline{128} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \overline{)64} \\ \underline{4} \\ 32 \\ \underline{64} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \overline{)32} \\ \underline{4} \\ 16 \\ \underline{32} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \overline{)16} \\ \underline{4} \\ 8 \\ \underline{16} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \overline{)8} \\ \underline{4} \\ 4 \\ \underline{8} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \overline{)4} \\ \underline{4} \\ 0 \end{array}$$

$$\frac{390625}{5} = 78125$$

$$= \frac{5}{3} \left(2 \left(\frac{390369}{390625} \right) \right) = 2 \left(\frac{130123}{78125} \right) = \frac{260246}{78125}$$

9. (10 pts) Find the sum of the infinite series $\sum_{k=1}^{\infty} 4 \cdot \left(\frac{3}{4}\right)^{k-1} = 4 + 4 \cdot \frac{3}{4} + 4 \cdot \left(\frac{3}{4}\right)^2 + \dots \approx 3.331148800$

$$4 \left(\frac{1}{1 - \frac{3}{4}} \right) = 4 \left(\frac{1}{\frac{1}{4}} \right) = 4 \cdot 4 = 16$$

Finance Formulas:

$$A = P \left(1 + \frac{r}{m} \right)^{mt} = P(1+r)^n$$

$$FV = R \left(\frac{(1+i)^n - 1}{i} \right)$$

10. (10 pts) What's the future value, in 10 years, of \$10,000 deposited into a savings account, earning 4.3% annual percentage rate *compounded daily?*

$$A = 10000 \left(1 + \frac{.043}{365} \right)^{(365)(10)} \approx \boxed{\$15372.19}$$

11. (10 pts) An annuity consists of monthly payments of \$600 into an account earning 8.4% annual interest, compounded monthly, for 10 years. There are two ways to ask this question:

First way: How much does JG Wentworth feel that this annuity is worth?

Second way: If the annuity described is actually your monthly loan payments, how much did you borrow in the first place?

$$A = FV$$

$$P(1+i)^n = R \left(\frac{(1+i)^n - 1}{i} \right)$$

$$2.309598381 P = 112251.2898$$

$$P = R \left(\frac{1 - (1+i)^{-n}}{i} \right)$$

$$P \approx \frac{112251.2898}{2.309598381} \approx 48602.08$$

$$= 600 \left(\frac{1 - \left(1 + \frac{.084}{12} \right)^{-12(10)}}{\frac{.084}{12}} \right) \approx$$

Check: \$72,000 total paid

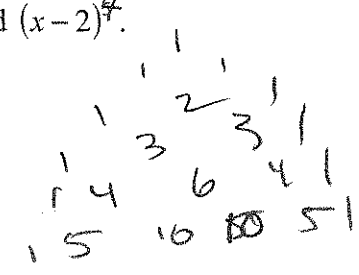
\$48,602.08

Bonus (10 pts) Use Pascal's Triangle (Binomial Theorem!) to help you expand $(x-2)^4$.

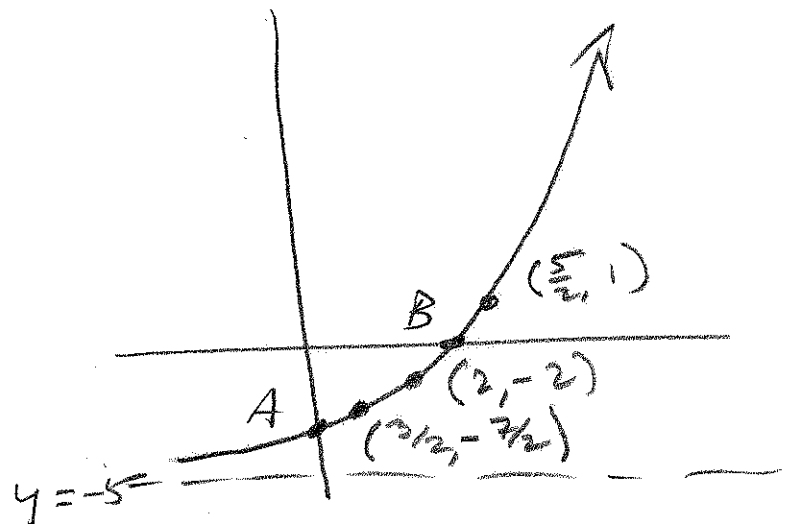
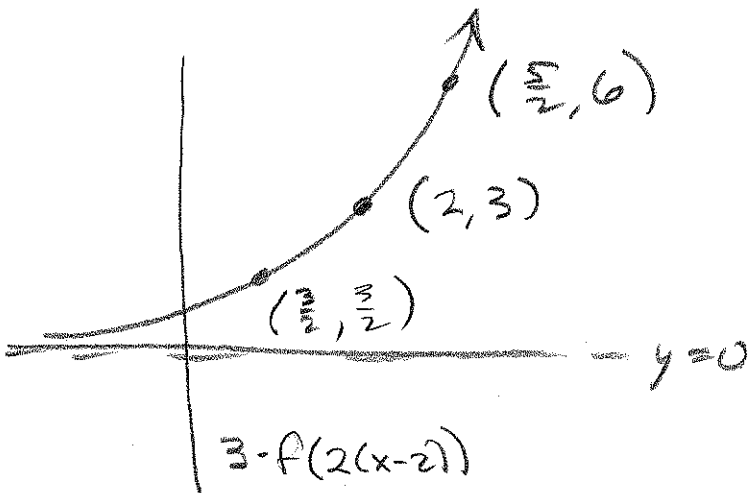
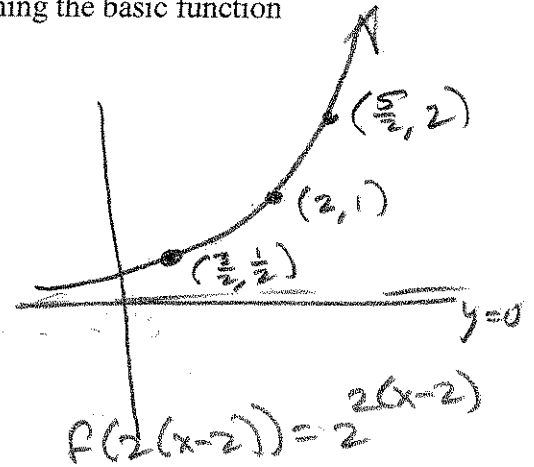
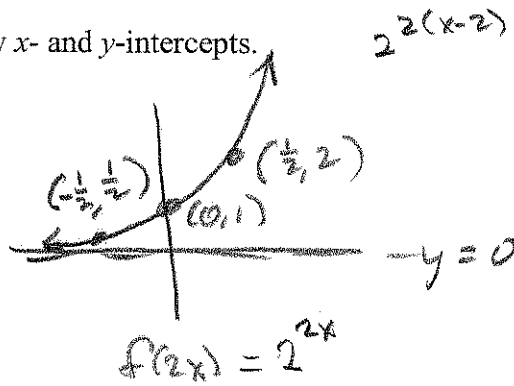
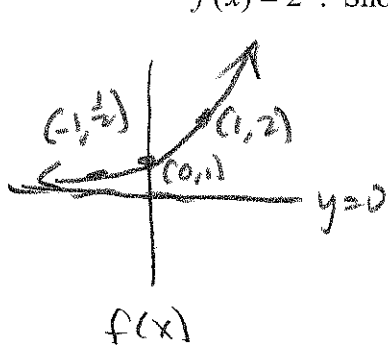
Expanding without using this technique will not earn any points.

$$x^4 + 4(x)^3(-2) + 6(x)^2(-2)^2 + 4(x)(-2)^3 + (-2)^4$$

$$= x^4 - 8x^3 + 24x^2 - 32x + 16$$



12. (15 pts) Sketch the graph of $g(x) = 3 \cdot 2^{2x-4} - 5$ by transforming the basic function $f(x) = 2^x$. Show x- and y-intercepts.



$$A = (0, f(0)) = (0, -\frac{73}{5})$$

$$\begin{aligned} & 3 \cdot 2^{2(0)-4} - 5 \\ &= 3 \cdot 2^{-4} - 5 \\ &= \frac{3}{16} - 5 \\ &= \frac{3-80}{16} \\ &= -\frac{77}{16} = -4.8125 \end{aligned}$$

$$B: 3 \cdot 2^{2x-4} - 5 = 0$$

$$\begin{aligned} 3 \cdot 2^{2x-4} &= 5 \\ 2^{2x-4} &= \frac{5}{3} \\ 2x-4 &= \log_2\left(\frac{5}{3}\right) \\ 2x &= 4 + \log_2\left(\frac{5}{3}\right) \\ x &= \frac{4 + \log_2\left(\frac{5}{3}\right)}{2} \end{aligned}$$

$$B = \left(\frac{4 + \log_2\left(\frac{5}{3}\right)}{2}, 0 \right)$$

$\approx (2.3685, 0)$