MAT 121 Fall, 2011
150 Points
Test 3-4, Chapters 3-4 Name KEY

1. (5 pts) Form a polynomial in factored form with *real* coefficients with the given zeros and degree. Please do not expand the polynomial.

Zeros: -3, multiplicity 2; 5, multiplicity 2. Degree 4.

$(x+3)^2(x-5)^2$

2. (5 pts) Expand $(x-4-2i)(x-4+2i)$

$x^2-4x+2ix-4x+16-8i-2ix+8i-4i^2$

$= x^2-8x+16+4 = x^2-8x+20$

3. (10 pts) Use synthetic division to find $P(2)$ if $P(x)=3x^4-2x^2+5x+1$.

$(x-2)(3x^3+6x^2+10x+25)+51$

2	3	0	-2	5	1
		6	12	20	50
	3	6	10	25	51 = P(2)

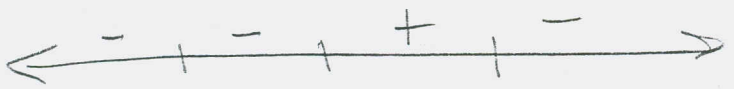
4. (10 pts) Divide $f(x)=2x^4-x^3+3x^2-4$ by $d(x)=x^2+1$. Then write the result in the form *Dividend* = *Divisor* · *Quotient* + *Remainder*.

$$\begin{array}{r} 2x^2-x+1 \\ x^2+1 \overline{)\,2x^4-x^3+3x^2+0x-4} \\ -(2x^4\qquad +2x^2) \\ \hline -x^3+x^2+0x-4 \\ -(-x^3\qquad -x) \\ \hline x^2+x-4 \\ -(x^2\qquad +1) \\ \hline x-5 \end{array}$$

$f(x) = (x^2+1)(2x^2-x+1)+x-5$

5. Solve the inequalities:

a. (5 pts) $-4(x-1)^2(x-2)(x-4)^3 \geq 0$



$x \in [2,4] \cup \{1\}$

b. (5 pts) $\dfrac{-4(x-1)^2}{(x-2)(x-4)^3} \geq 0$

$x \in (2,4) \cup \{1\}$

6. (10 pts) Find all intercepts, asymptotes and holes, and then sketch the graph of

$$f(x) = \frac{x^3 - 3x^2 - 4x + 12}{x^3 + 2x^2 - 5x - 6} = \frac{(x-2)(x+2)(x-3)}{(x-2)(x+3)(x+1)}$$

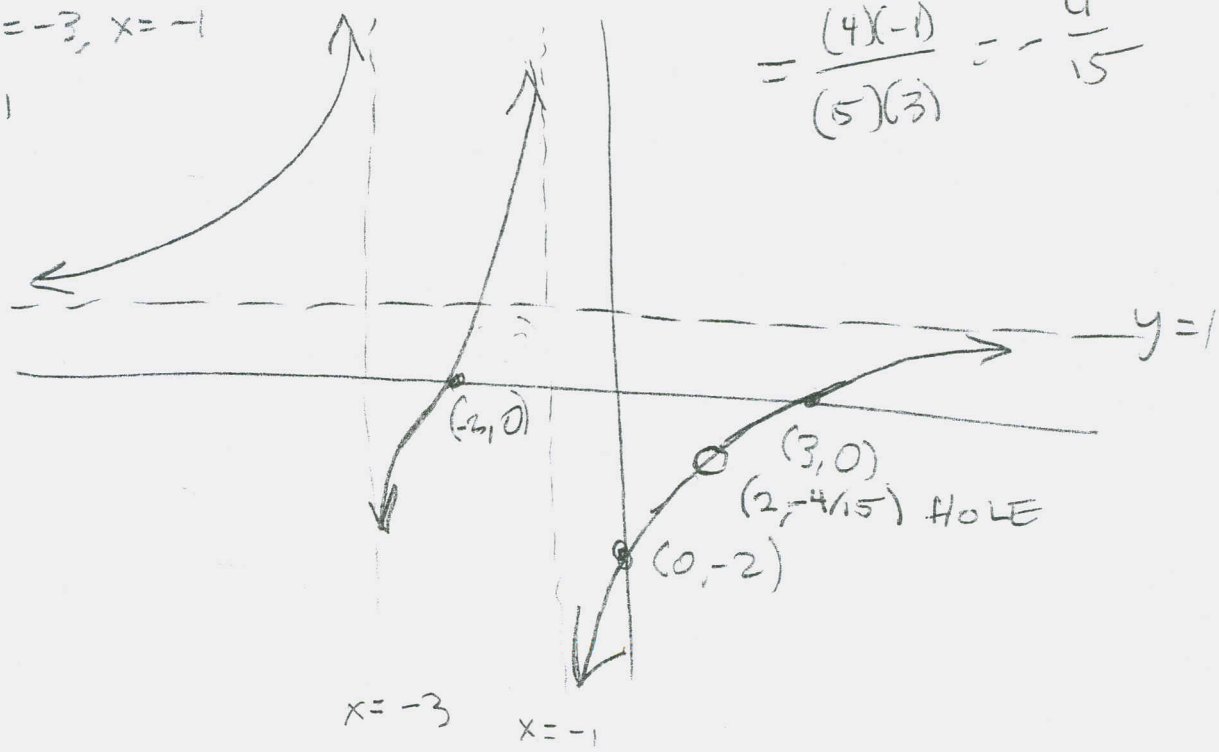
Hole: $x = 2$

Zeros: $x = -2, 3$

V.A.: $x = -3, x = -1$

EB: $y = 1$

Hole: $\dfrac{(2+2)(2-3)}{(2+3)(2+1)} = \dfrac{(4)(-1)}{(5)(3)} = -\dfrac{4}{15}$



7. Solve the equations for x:

a. (10 pts) $25^{2x-3} = 5^{x+1}$

$(5^{-2})^{2x-3} = 5^{x+1}$

$2(2x-3) = x+1$

$4x-6 = x+1$

$3x = 7$

$x = \frac{7}{3}$

b. (10 pts) $3 \cdot 2^{x+1} - 4 = 0$

$3 \cdot 2^{x+1} = 4$

$2^{x+1} = \frac{4}{3}$

$x+1 = \log_2\left(\frac{4}{3}\right)$

$x = -1 + \log_2\left(\frac{4}{3}\right)$

$= -1 + \frac{\ln(4/3)}{\ln(2)} \approx -.5849625007$

c. (10 pts) $-2\log_3(-2x+6) = 0$

$\log_3(-2x+6) = 0$

$-2x+6 = 3^0 = 1$

$-2x = -5$

$x = \frac{5}{2}$

d. (10 pts) $P\left(1+\frac{.06}{12}\right)^{12x} = 3P$

$\left(1+\frac{.06}{12}\right)^{12x} = 3$

$12x \log\left(1+\frac{.06}{12}\right) = \log(3)$

$x = \frac{\log(3)}{12\log\left(1+\frac{.06}{12}\right)}$

≈ 18.35594227

e. (10 pts) $\ln(x-2)+\ln(x+1) = \ln(4)$

$\ln((x-2)(x+1)) = \ln(4)$

$(x-2)(x+1) = 4$

$x^2 - x - 2 = 4$

$x^2 - x - 6 = 0$

Note to self: Guess and check or cheat was common, here.

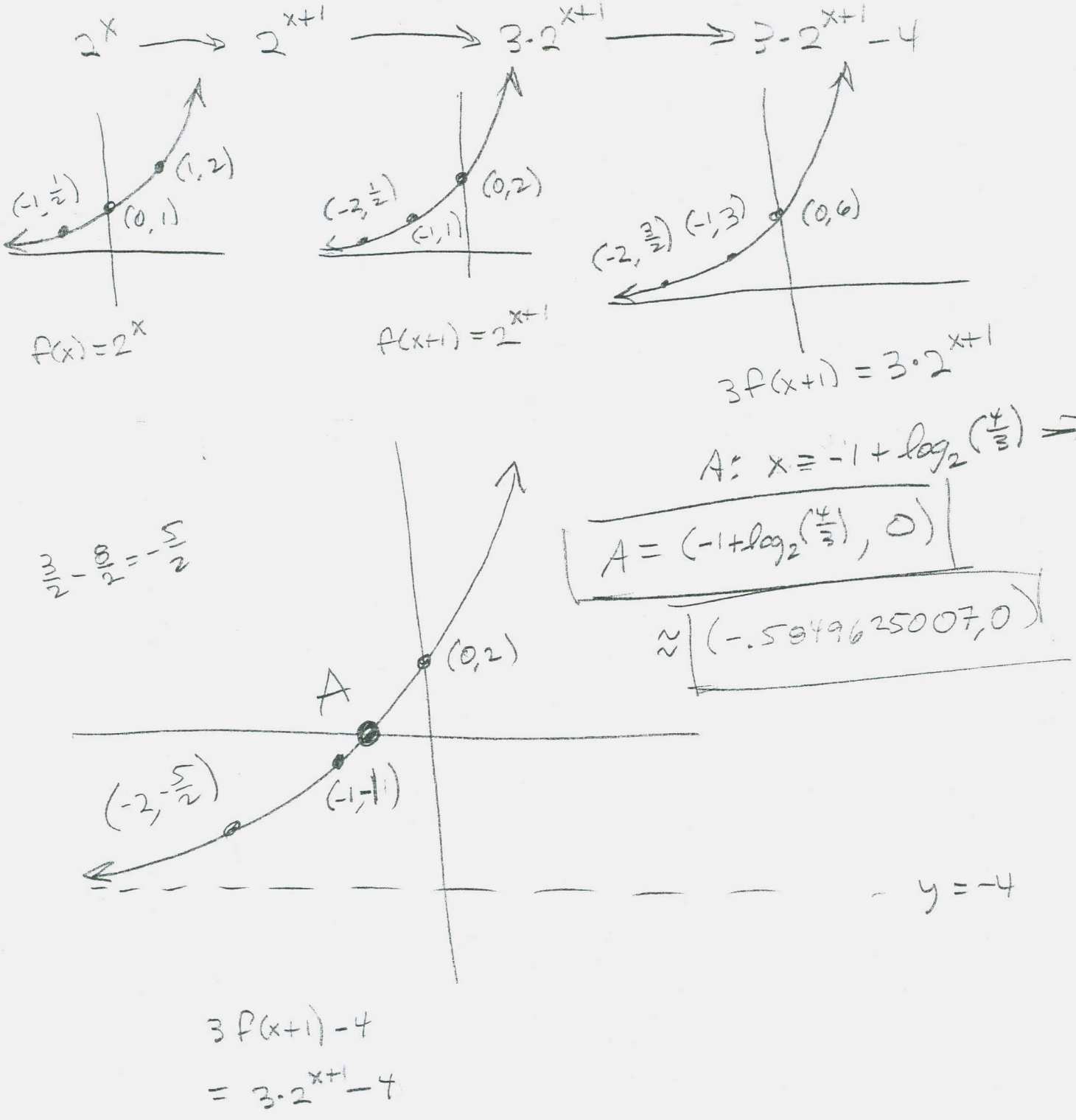
$(x-3)(x+2) = 0$

$x = -2, 3$ but

$x = -2 \notin$ Domain, so

$x = 3$

8. (20 pts) Sketch the graph of $g(x) = 3 \cdot 2^{x+1} - 4$ by transforming the function $f(x) = 2^x$. Use $x = -1$, $x = 0$, and $x = 1$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$. Your final graph should also show the y-intercept and the x-intercept.



$A: \; x = -1 + \log_2\left(\frac{4}{3}\right) \Rightarrow$

$A = \left(-1 + \log_2\left(\frac{4}{3}\right), 0\right)$

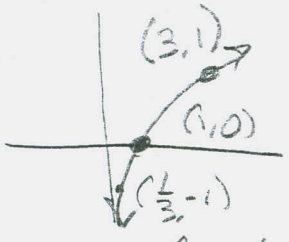
$\approx (-.5849625007, 0)$

9. (20 pts) Sketch the graph of $h(x) = -2\log_3(-2x+6)$ by transforming the function $f(x) = \log_3(x)$. Show the points on the first graph that correspond to $x = \frac{1}{3}, 1, 3$, and show how they are moved around by each step in the transformations to $h(x)$. Be sure to show the x-intercept on the final graph.

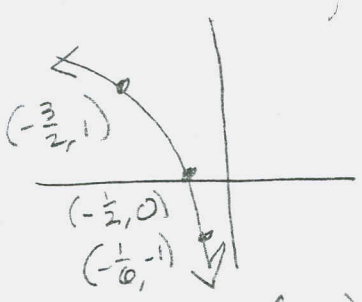
$-2x+6 = -2(x-3)$

$\log_3(x) \longrightarrow \log_3(-2x) \longrightarrow \log_3(-2(x-3)) \longrightarrow$

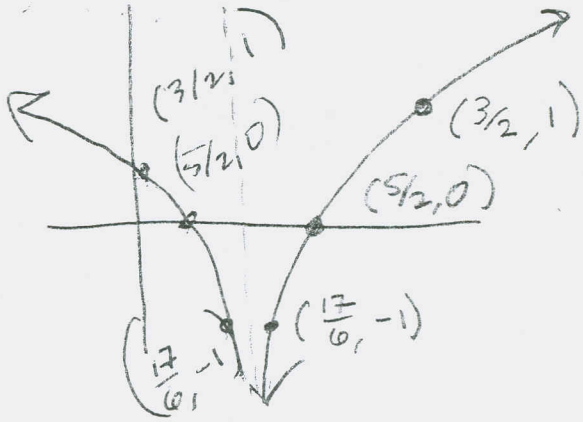
$-2\log_3(-2(x-3))$



$f(x) = \log_3(x)$



$f(-2x) = \log_3(-2x)$



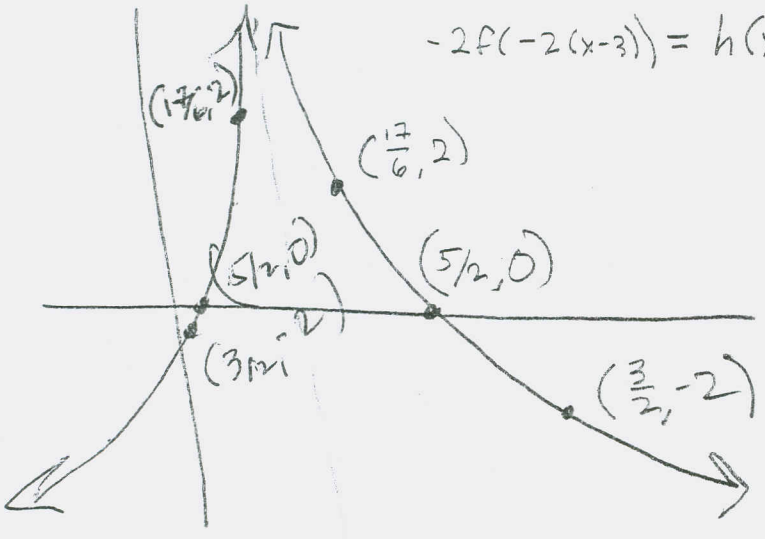
$f(-2(x-3)) = \log_3(-2(x-3))$

$-\frac{1}{6} + \frac{18}{6} = \frac{17}{6}$

$-\frac{1}{2} + \frac{6}{2} = \frac{5}{2}$

$-\frac{3}{2} + \frac{6}{2} = \frac{3}{2}$

$-2f(-2(x-3)) = h(x)$



10. (10 pts) Suppose the half-life of a radioactive substance is 500 years. How old is a fire pit, if there is only 12% of the naturally occurring radioactive substance present in a charcoal sample taken from the pit?

$P_0 e^{500k} = \frac{1}{2} P_0$

$e^{500k} = \frac{1}{2}$

$500k = \ln\left(\frac{1}{2}\right) = -\ln(2)$

$k = \frac{-\ln(2)}{500}$

AGE: $P_0 e^{kt} = .12 P_0$

$e^{kt} = .12$

$kt = \ln(.2)$

$t = \frac{\ln(.2)}{k} = \frac{-500\ln(.2)}{\ln(2)}$

$\approx$ 1,529.446845 yrs