1. (10 pts) Solve $x^2 - 5x + 4 = 0$ by factoring.

$$(x-4)(x-1) = 0$$

$$x \in \{1, 4\}$$

2. (10 pts) Solve $x^2 - 5x + 4 = 0$ by completing the square. $x^2 - 6x + 7 = 0$

$$x^2 - 5x + \left(\tfrac{5}{2}\right)^2 = -4 + \tfrac{25}{4}$$

$$\left(x - \tfrac{5}{2}\right)^2 = \tfrac{-16+25}{4} = \tfrac{9}{4}$$

$$x - \tfrac{5}{2} = \pm \tfrac{3}{2}$$

$$x = \frac{5 \pm 3}{2} \quad \begin{cases} \tfrac{8}{2} = 4 \\ \tfrac{2}{2} = 1 \end{cases}$$

$$x \in \{1, 4\}$$

$$x^2 - 6x = -7$$

$$x^2 - 6x + 3^2 = -7 + 9$$

$$(x-3)^2 = 2$$

$$x - 3 = \pm\sqrt{2}$$

$$x = 3 \pm \sqrt{2}$$

Be clear on where the $\pm$ comes from

$$x \in \{3-\sqrt{2}, 3+\sqrt{2}\}$$

$\tfrac{5}{16}$
$\tfrac{9}{144}$

3. Compute the discriminant for each of the following. By this, tell me what kind of solutions there are, and how many there are.

a. (5 pts) $4x^2 - 5x - 9 = 0$

2 real (and rational) sol'ns

$$a = 4, b = -5, c = -9$$

$$b^2 - 4ac = 25 - 4(4)(-9)$$

$$= 25 + 144 = 169 > 0$$

b. (5 pts) $4x^2 - 5x + 9 = 0$ (Anything special about this one?)

Two nonreal sol'ns

$$a = 4, b = -5, c = 9$$

$$b^2 - 4ac = 25 - 4(4)(9)$$

$$= 25 - 144 = -119 < 0$$

c. (5 pts) $4x^2 - 12x - 9 = 0$ (Anything special about this one?)

$$b^2 - 4ac = 144 - 4(4)(-9)$$

$$= +288$$

Two

25

4. (10 pts) Solve the equation $2.5x^2 + 7.3x - 12.1 = 0$ correct to 4 decimal places.

$a = 2.5, \ b = 7.3, \ c = -12.1$

$b^2 - 4ac = (7.3)^2 - 4(2.5)(-12.1)$

$= 174.29$

$x = \dfrac{-7.3 \pm \sqrt{174.29}}{2(25)}$

$\nearrow \ 1.180037876/$

$\searrow \ -4.100378761$

$x \in \{1.1804, -4.1004\}$

(left margin, vertical): 1.18037788 -4.1003788

5. (5 pts) Jill can finish the paint job by herself in 12 hours and Jack can finish the paint job by himself in 9 hours. How long will it take Jack and Jill to finish the paint job, if they work together?

$\dfrac{2|12 \quad 3|9}{2|6 \qquad 3}$
$\dfrac{}{3}$

$x = $ the amt of time Jill spends. (in hrs)

$LCD = 2 \cdot 2 \cdot 3 \cdot 3 \cdot X$
$= 36X$

$\left[\dfrac{1}{12} + \dfrac{1}{9} = \dfrac{1}{x} \right] 36x$

$3x + 4x = 36$

$7x = 36$

$\boxed{x = \dfrac{36}{7} \ hr}$

$\dfrac{36}{7} = 5.14285714$

(#6) $x = $ amt. of time for Jack (in hrs)

version #2 for #6 :

$\dfrac{1}{12}(x-3) + \dfrac{1}{9}x = 1$

$3(x-3) + 4x = 36$

$3x - 9 + 4x = 36$

$7x = 45$

$x = \dfrac{45}{7}$

$\dfrac{45}{7} = 6.4285714$

$7am + 6hrs + \dfrac{3}{7}hrs$

$1pm + \left(\dfrac{3}{7}\right)(60)$

$= 1pm + 25.7 min$

$\approx 1:26 pm$

6. (5 pts) Follow-up to the previous problem. Suppose Jill gets a late start, and shows up to work at 10 a.m., and Jack has been there since 7 a.m. What time will they finish the job?

Let $x = $ amt of time jill spends. Then (in hrs)

$\left[\dfrac{1}{12}x + \dfrac{1}{9}(x+3) = 1 \right] 36$

$3x + 4(x+3) = 36$

$3x + 4x + 12 = 36$

$7x + 12 = 36$

$7x = 24$

$\boxed{x = \dfrac{24}{7}}$

$10 am + \dfrac{24}{7} hrs$ is the idea.

$\dfrac{24}{7} hrs = 3\dfrac{3}{7} hrs$

$\left(\dfrac{3}{7} hrs\right)\left(\dfrac{60 min}{1 hr}\right) \approx 25.7142$
≈ 26

$10 am + 3hrs + 26 min. \approx$

$\boxed{1:26 pm}$

(bottom left): 20

7. (10 pts) Joe wants to mix 15% alcohol with 10 gallons of 25% alcohol, to obtain a mixture of 22% alcohol. How much 15% alcohol should he use, and what is the volume of the final mixture?

$x = $ amt of 15% alcohol (gallons)

15%	x	$.15x$
25%	10	$.25(10)$
TOT: 22%	$x+10$	$.22(x+10)$

$.15x + .25(10) = .22(x+10)$

$.15x + 2.5 = .22x + 2.2$

Amt. Pure $= .15x + .25(10) = .22(x+10)$

$-.07x = -.3$

$x = \dfrac{.3}{.07} = \dfrac{3}{.7} = \dfrac{30}{7} \approx 4.285714286$

≈ 4.286 gallons of 15%

TOTAL Volume $\approx 10 + 4.286$

≈ 14.286 gallons of 15% alcohol

8. (10 pts) Solve $\dfrac{x-12}{3-x} = \dfrac{x+16}{x+5}$.　　$LCD = (3-x)(x+5)$

$(x-12)(x+5) = (x+16)(3-x)$

$x^2 - 7x - 60 = 3x - x^2 + 48 - 16x$

$x^2 - 7x - 60 = -x^2 - 13x + 48$

$2x^2 + 6x - 108 = 0$

$x^2 + 3x - 54 = 0$

$(x+9)(x-6) = 0$

$x = -9 \ OR \ x = 6$

$x \in \{-9, 6\}$

Check:

$\dfrac{-9-12}{3+9} \overset{?}{=} \dfrac{-9+16}{-9+5}$

$-\dfrac{7}{4} = \dfrac{-21}{12} = \dfrac{7}{-4} \ \checkmark$

$\dfrac{6-12}{3-6} \overset{?}{=} \dfrac{6+16}{6+5}$

$\dfrac{-6}{-3} = 2 \overset{?}{=} \dfrac{22}{11} \ \checkmark$

20

9. (5 pts) Solve the compound inequality $2x - 3 > 5$ or $5 - 3x > 11$. State the final answer in

$$2x - 3 > 5 \qquad\qquad 5 - 3x > 11$$
$$2x > 8 \qquad\qquad -3x > 6 \qquad \rightarrow \text{Set-builder}$$
$$\{x \mid x > 4 \qquad \text{OR} \qquad x < -2\}$$

$$(-\infty, -2) \cup (4, \infty) \qquad \rightarrow \text{Interval}$$

Solve the absolute value inequalities.

10. (5 pts) $|7x + 2| > 5$

$$7x + 2 > 5 \quad \text{OR} \quad 7x + 2 < -5$$
$$7x > 3 \qquad\qquad 7x < -7$$
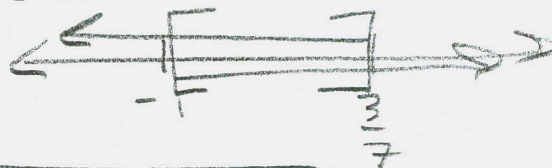$$\{x \mid x > \tfrac{3}{7} \quad \text{OR} \quad x < -1\}$$
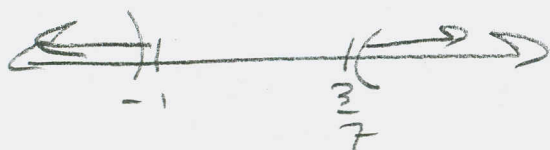
$$(-\infty, -1) \cup (\tfrac{3}{7}, \infty)$$

11. (5 pts) $|7x + 2| \leq 5$

$$7x + 2 \leq 5 \quad \text{AND} \quad 7x + 2 \geq -5$$
$$7x \leq 3 \qquad\qquad 7x \geq -7$$
$$\{x \mid x \leq \tfrac{3}{7} \quad \text{and} \quad x \geq -1\}$$

$$[-1, \tfrac{3}{7}]$$

Solve the degenerate cases. If you run out of room, you're doing it wrong.

12. (5 pts) $|2 - 7x| > -3$

$$\mathbb{R} = (-\infty, \infty)$$
$$= \{x \mid x \text{ is real}\}$$

13. (5 pts) $|2 - 7x| < -3$ Nevah!

$$\varnothing$$