MAT 121 Spring, 2011

Test 3, Chapter 3 80 Points Name KEY

1. In each of the following, form a polynomial with *real* coefficients that has the given zeros and degree. Please do not expand the polynomial.

   a. (5 pts) Zeros: 2, multiplicity 2; - 3, multiplicity 2. Degree 4.

   $(x-2)^2(x+3)^2$

   $x^4+2x^3-11x^2-12x+36$

   b. (5 pts) Zeros: 2, multiplicity 1; - 3, multiplicity 2; 7 - 8*i*, multiplicity 1. Degree 5.

   $(x-2)(x+3)^2(x-(7-8i))(x-(7+8i))$

2. (5 pts) Expand $(x+(2-3i))(x+(2+3i))$

   $x^2+(2+3i)x+(2-3i)x+(2^2+3^2)$

   $= x^2+2x+3ix+2x-3ix+13 = x^2+4x+13$

3. (5 pts) Use synthetic division to find $P(3)$ if $P(x)=2x^5-3x^3+3x^2-4x+13$.

3	2	0	-3	3	-4	13
		6	18	45	144	420
	2	6	15	48	140	433 = P(3)

4. (5 pts) Divide $f(x)=x^4-3x^3+2x^2+5$ by $f(x)=x^2-2$.

   $x^2-2 \;\overline{)\; x^4-3x^3+2x^2+0x+5}$

   x^2-3x+4 R $-6x+13$

   $-(x^4 \quad -2x^2)$

   $-3x^3+4x^2+0x+5$

   $-(-3x^3 \quad +6x)$

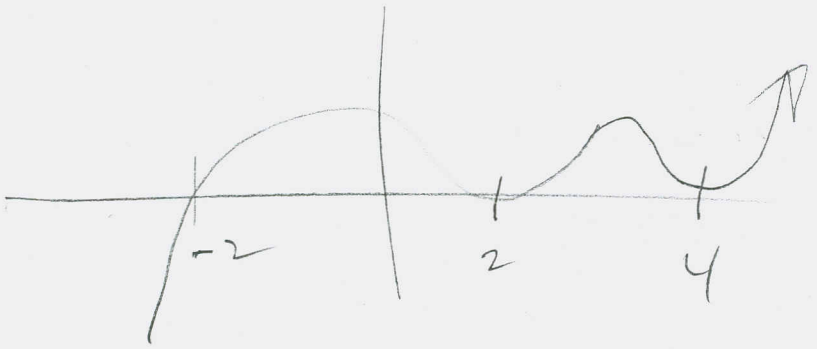
   $4x^2-6x+5$

   $-(4x^2 \quad -8)$

   $-6x+13$

   $f(x)=(x^2-2)(x^2-3x+4)+(-6x+13)$

6. Use your sketch from the previous problem to help you solve the following inequalities. You might want to re-sketch it, below, just to have it on the same page.



a. (5 pts) $2(x-2)^2(x+2)(x-4)^2 \le 0$

$(-\infty, -2] \cup \{2, 4\}$

b. (5 pts) $\dfrac{2(x-2)^2}{(x+2)(x-4)^2} \le 0$

$(-\infty, -2) \cup \{2\}$

7. $x = 5$ is UB for $f(x) = x^4 - 5x^3 + 15x^2 - 5x - 26$

5	1	-5	15	-5	-26
		5	0	75	350
	1	0	15	70	324

Bottom row is nonnegative.

9. (10 pts) Suppose $R(x)=\dfrac{x^3-8x^2+x+42}{x^3-x^2-10x-8}$ can be factored into $\dfrac{(x-3)(x+2)(x-7)}{(x+2)(x-4)(x+1)}$.

(It can.) Sketch the graph of R showing all intercepts, asymptotes and holes (if any).

1pt $\mathcal{D} = \mathbb{R} \setminus \{-2, -1, 4\}$

$R(0) = \dfrac{(-3)(-7)}{(-4)(1)}$

Hole: $x = -2$

$R^*(-2) = \dfrac{(-2-3)(-2-7)}{(-2-4)(-2+1)} = \dfrac{(-5)(-9)}{(-6)(-1)} = \dfrac{45}{6} = \dfrac{15}{2}$

1pt Hole: $(-2, \frac{15}{2})$

2pts V.A.: $x = -1, x = 4$

1pt H.A.: $\frac{x^3}{x^3} = 1 = y$

2pts x-int: $(3,0), (7,0)$

1pt y-int: $(0, -\frac{21}{4})$

At $x = -1$: cross

$x = 4$: cross

$x = 3$: cross

$x = 7$: cross