MAT 121 100 Points Fall, 2010

Test 5, Chapter 5 Name Key

1. (10 pts) Determine whether (5, 2) is in the solution set for the following system:

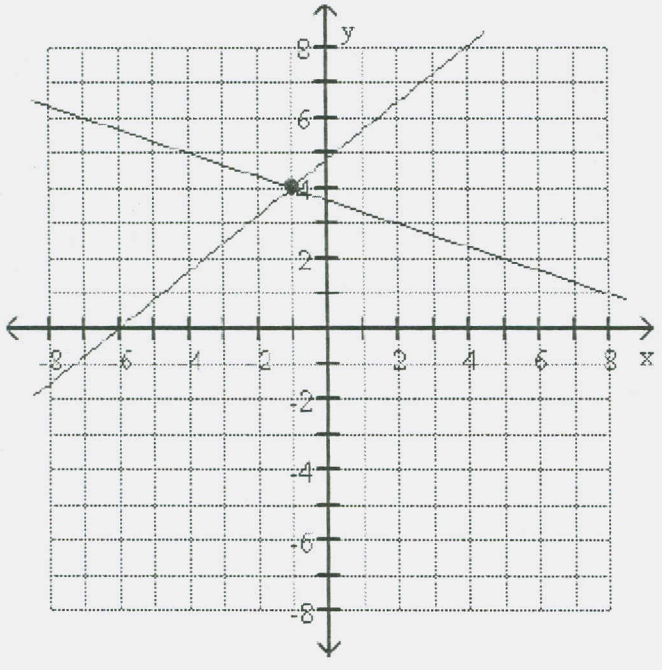
$x + y = 7$
$x - y = 3$

$5 + 2 = 7$ ✓
$5 - 2 = 3$ ✓

Yes

2. (10 pts) Solve the following system by inspecting the graph of the equations:

$x + 3y = 11$
$4x - 5y = -24$



$\{(-1, 4)\}$

3. (10 pts) Solve the following system by any method:

$x + y = 2$
$x - y = -12$

$2x = -10$

$x = -5$

$-5 + y = 2$

$y = 7$

$\{(-5, 7)\}$

214
6
84

84
45
129

4. (10 pts) Solve the following system by addition (or by Gaussian Elimination).

$$x - 6y = -14$$
$$-6x - 7y = -45$$

$$\left[\begin{array}{cc|c} 1 & -6 & -14 \\ -6 & -7 & -45 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -6 & -14 \\ 0 & -43 & -129 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & -6 & -14 \\ 0 & 1 & 3 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 3 \end{array}\right]$$

$\{(4, 3)\}$

86
43
129

$$\left[\begin{array}{cc} 1 & -6 \\ -6 & -7 \end{array}\right]\left[\begin{array}{c} 4 \\ 3 \end{array}\right] = \left[\begin{array}{c} -14 \\ -45 \end{array}\right]$$

24
45
69

5. (10 pts) Classify the systems as independent, inconsistent or dependent. You should be able to decide, by inspection (on these 2x2s).

a. $x + y = 2$
$x + y = 7$

Inconsistent

b. $x + y = 2$
$3x + 3y = 6$ $= 3R1$

Dependent

c. $x + y = 2$
$x + 2y = 7$

Independent

6. (10 pts) Solve the following word problem. (Take it all the way to solution.)

A shopkeeper orders 42 pounds of cashews and peanuts. If he orders 12 less pounds of cashews, how many pounds of peanuts did he order?

$x = C =$ # of lbs of cashews $\quad C = P - 12$

$y = P =$ " " " " nuts

$y + x = 42$

$P + C = 42$

$x = y - 12$

$C = P - 12$

$P + P - 12 = 42$

$2P = 54$

$P = 27 \Rightarrow$

$C = P - 12 = 27 - 12 = 15$

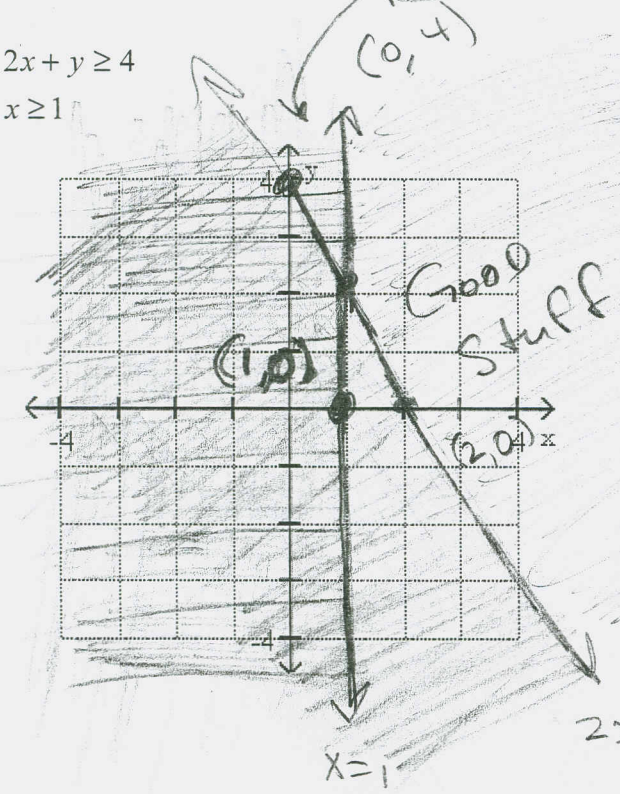
$(C, P) = (15, 27)$

He ordered 27 lbs of peanuts

7. (10 pts) Graph the solution set of the system.

$2x + y \geq 4$

$x \geq 1$



$2x + y = 4$

x	y
0	4
2	0

$0 \geq 4$? No

(0,0) Bad

8. (10 pts) Solve the system of equations with matrices and Gaussian Elimination:

$x+y+z=6$
$x-y+5z=20$
$4x+y+z=0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & 5 & 20 \\ 4 & 1 & 1 & 0 \end{array}\right]$$

$$\begin{matrix} \\ -R1+R2 \\ -4R1+R3 \end{matrix}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & 4 & 14 \\ 0 & -3 & -3 & -24 \end{array}\right]$$

$$\begin{matrix} \\ -\frac{1}{2}R2 \\ -\frac{1}{3}R3 \end{matrix}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -7 \\ 0 & 1 & 1 & 8 \end{array}\right]$$

$$\begin{matrix} -R2+R1 \\ \\ -R2+R3 \end{matrix}\left[\begin{array}{ccc|c} 1 & 0 & 3 & 13 \\ 0 & 1 & -2 & -7 \\ 0 & 0 & 3 & 15 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 13 \\ 0 & 1 & -2 & -7 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\begin{matrix} -3R3+R1 \\ 2R3+R2 \\ \\ \end{matrix}\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$\{(-2,3,5)\}$

$$\left[\begin{array}{ccc} 1 & 1 & 1 \\ 1 & -1 & 5 \\ 4 & 1 & 1 \end{array}\right]\left[\begin{array}{c} -2 \\ 3 \\ 5 \end{array}\right] = \left[\begin{array}{c} 6 \\ 20 \\ 0 \end{array}\right] \checkmark$$

$y \approx .1924500897$

9. (10 pts) Solve the system of nonlinear equations:

$y = 27^x = 3^{3x}$

$x = \log_3(3y) \Rightarrow 3^x = 3y \Rightarrow \frac{1}{3}\cdot 3^x = y \Rightarrow$

$3^{-1}\cdot 3^x = y \Rightarrow 3^{x-1} = y$. This gives

$3^{3x} = 3^{x-1}$

$3x = x-1$

$2x = -1$

$x = -\frac{1}{2}$

So $y = 3^{3(-\frac{1}{2})} = 3^{-\frac{3}{2}} = \frac{1}{3^{\frac{3}{2}}} = \frac{1}{\sqrt{3^3}} = \frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{3\sqrt{3}\sqrt{3}} = \frac{\sqrt{3}}{9} = y$

Check: $x = \log_3\left(3\cdot\frac{\sqrt{3}}{9}\right) = \log_3\left(\frac{3\cdot 3^{\frac{1}{2}}}{3^2}\right) = \log_3\left(\frac{3^{\frac{3}{2}}}{3^2}\right) = \log_3\left(3^{-\frac{1}{2}}\right) = -\frac{1}{2}$ ✓

$y = 27^{-\frac{1}{2}} = \frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{3\cdot 3} = \frac{\sqrt{3}}{9} = y$

10. (10 pts) Set up the following word problem (variables, system, augmented matrix). For 10 bonus points, *solve* the word problem.

Linda invests $25,000 for one year. Part is invested at 5%, another part at 6%, and the rest at 8%. The total income from all 3 investments is $1600. The combined income from the 5% and 6% investments is the same as the income from the 8% investment. Find the amount invested at each rate.

Let x = the amt (in $) Linda invests @ 5%

$y =$ " " " " 6%

$.05x + .06y = .08z$ $z =$ " " " " 8%

Then $x + y + z = 25000$

$.05x + .06y + .08z = 1600$

$.05x + .06y - .08z = 0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 25000 \\ .05 & .06 & .08 & 1600 \\ .05 & .06 & -.08 & 0 \end{array}\right]$$

$x = 10000$ $z = 10000$

$y = 5000$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 25000 \\ .05 & .06 & .08 & 1600 \\ .05 & .06 & -.08 & 0 \end{array}\right]$$

$\cdot R3$

$-R3+R2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 25000 \\ .05 & .06 & -.08 & 0 \\ 0 & 0 & .16 & 1600 \end{array}\right]$$

$-.05R1+R2$

$\frac{1}{.16}R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 25000 \\ 0 & .01 & -.13 & -1250 \\ 0 & 0 & 1 & 10000 \end{array}\right]$$

$\frac{1}{.01}R2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 25000 \\ 0 & 1 & -13 & -125000 \\ 0 & 0 & 1 & 10000 \end{array}\right]$$

$z = 10000$

$y - 13z = -125000$

$y - 13(10000) = -125000$

$y - 130000 = -125000$

$y = 5000$

$x + 5000 + 10000 = 25000$

$x + 15000 = 25000$

$x = 10000$

$\{(10000, 5000, 10000)\}$