MAT 121 80 Points Fall, 2010
Test 3, Chapter 3 Name_____

1. In each of the following, form a polynomial with *real* coefficients that has the given zeros and degree. Please do not expand the polynomial.

   a. (5 pts) Zeros: - 4, multiplicity 2; 2, multiplicity 3. Degree 5.

$f(x) = (x+4)^2(x-2)^3$

   b. (5 pts) Zeros: 2, multiplicity 1; 5, multiplicity 2; 7 - 8*i*, multiplicity 1. Degree 5.

$f(x) = (x-2)(x-5)^2(x-(7-8i))(x-(7+8i))$

2. (5 pts) Expand $(x-(3+6i))(x-(3-6i))$

$= x^2 - (3-6i)x - (3+6i)x + (3+6i)(3-6i)$

$= x^2 - 3x + 6ix - 3x - 6ix + 3^2 + 6^2 = \boxed{x^2 - 6x + 45}$

3. (5 pts) Use synthetic division to find $P(2)$ if $P(x) = x^4 - 5x^3 + 11x^2 - 12x + 13$.

$$\begin{array}{c|ccccc} 2 & 1 & -5 & 11 & -12 & 13 \\ & & 2 & -6 & 10 & -4 \\ \hline & 1 & -3 & 5 & -2 & 9 \end{array}$$

$\boxed{P(2) = 9}$

4. (5 pts) Divide $f(x) = 2x^4 - 3x^3 + x - 3$ by $f(x) = x^2 - 1$

$$\begin{array}{r} 2x^2 - 3x + 2 \text{ r } -2x - 1 \\ x^2 - 1 \overline{)\, 2x^4 - 3x^3 + 0x^2 + x - 3} \\ -(2x^4 \quad - 2x^2) \\ \hline -3x^3 + 2x^2 + x - 3 \\ -(-3x^3 + \quad 3x) \\ \hline 2x^2 - 2x - 3 \\ -(2x^2 \quad - 2) \\ \hline -2x - 1 \end{array}$$

5. Let $f(x) = 2(x-1)^2(x+4)(x-5)^3$.

a. (5 pts) List each real zero and its multiplicity. Determine whether the graph of $f(x)$ touches or crosses the x-axis at each x-intercept.

$x=1$, $m=2$ touch

$x=-4$, $m=1$ cross

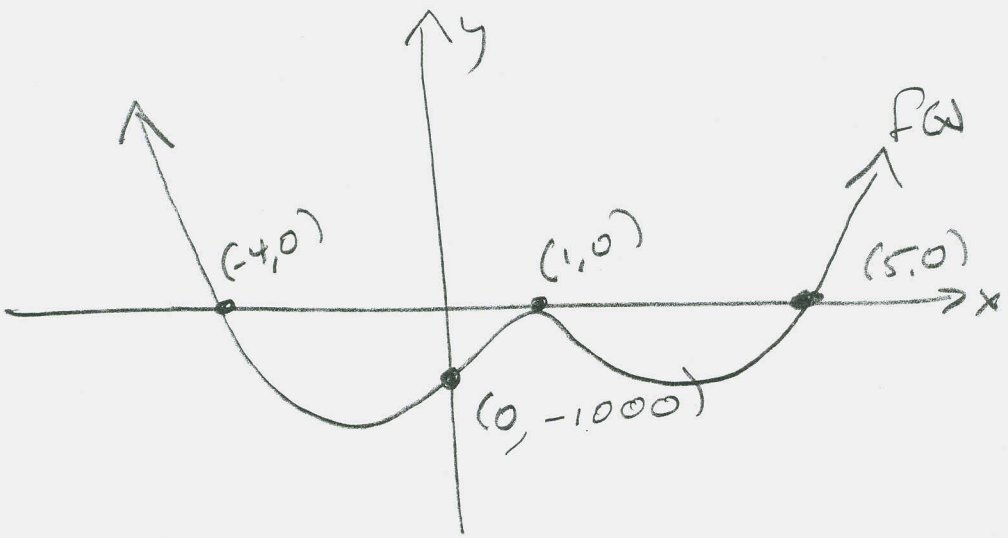
$x=5$, $m=3$ cross

b. (5 pts) Find the following limits:

i. $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} 2(x)^2(x)(x)^3 = \lim_{x\to\infty} 2x^6 = +\infty$

ii. $\lim_{x\to-\infty} f(x) = \ldots = +\infty$

c. (5 pts) Use your work, above, to help you sketch the graph of $f(x)$, showing all intercepts (including the y-intercept).



$f(0) = 2(-1)^2(4)(-5)^3 = (8)(-125) = -4(250) = -2(500)$

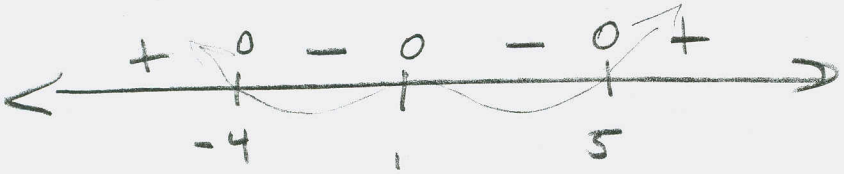
$= -1000$

15

6. Solve the inequalities (Hint: You already laid the foundations for *both* of these in the previous problem.).

(10 pts)

a. (5 pts) $2(x-1)^2(x+4)(x-5)^3 \geq 0$



$(-\infty, -4] \cup [5, \infty) \cup \{1\}$

$\frac{(x+4)(x-1)^2}{(x-5)^3}$

b. (5 pts) $\frac{(x-1)^2}{(x+4)(x-5)^3} \geq 0$ (Hint: This one differs only *slightly* from the previous one.)

$(-\infty, -4] \cup (5, \infty) \cup \{1\}$

Can't let $x=5$. Otherwise, the same as previous.

7. (10 pts) Find *all* the zeros of $f(x) = x^4 - 5x^3 + 15x^2 - 5x - 26$. Write f in factored form, using the zeros you find.

Descartes: 3 or 1 pos.

$f(-x) = x^4 + 5x^3 + 15x^2 + 5x - 26$

1 negative.

$\frac{p}{q} = \pm 1, \pm 2, \pm 13, \pm 26$

$-1 \mid 1 \quad -5 \quad 15 \quad -5 \quad -26$

$\qquad\quad -1 \quad 6 \quad -21 \quad 26$

$\qquad 1 \quad -6 \quad 21 \quad -26 \quad 0$ yes

$(x+1)(x^3 - 6x^2 + 21x - 26)$

$1 \mid 1 \quad -6 \quad 21 \quad -26$

$\qquad\quad 1 \quad -5$ No

$\qquad 1 \quad -5 \quad 16$

$2 \mid 1 \quad -6 \quad 21 \quad -26$

$\qquad\quad 2 \quad -8 \quad 26$

$\qquad 1 \quad -4 \quad 13 \quad 0$ Yes

$(x+1)(x-2)(x^2 - 4x + 13)$

$x^2 - 4x + 13 = 0$

$x^2 - 4x = -13$

$x^2 - 4x + 2^2 = -13 + 4$

$(x-2)^2 = -9$

$x - 2 = \pm\sqrt{-9} = \pm 3i$

$x = +2 \pm 3i$

Zeros:

$x = -1, 2, +2 \pm 3i$

FACTORED FORM:

$(x+1)(x-2)(x-(+2+3i))(x-(+2-3i))$

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8. (10 pts) Graph the function $R(x)=\dfrac{2x^3-3x^2-2x+3}{x^3-4x^2+x+6}=\dfrac{(x-1)(2x-3)(x+1)}{(x+1)(x-2)(x-3)}$. Key features are asymptotes, holes (if any) and intercepts. I was kind enough to factor it for you.

$D=\{x \mid x\neq -1,\ x\neq 2,\ \text{and } x\neq 3\}$

Hole @ $x=-1$

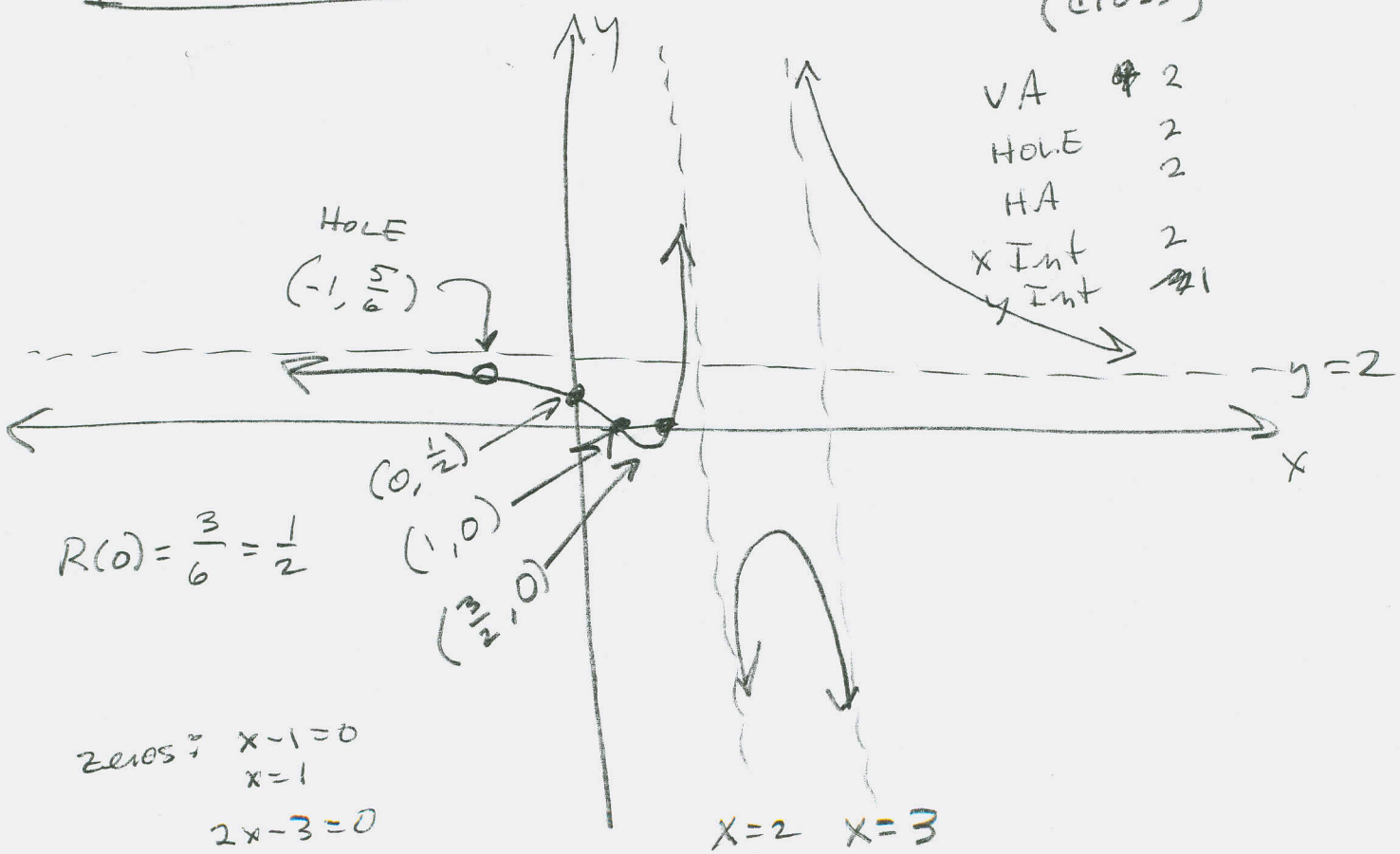
$R^*(-1)=\dfrac{(-1-1)(2(-1)-3)}{(-1-2)(-1-3)}=\dfrac{(-2)(-2-3)}{(-3)(-4)}=\dfrac{(-2)(-5)}{12}$

$=\dfrac{10}{12}=\dfrac{5}{6}$ ⇝ $\left(-1,\dfrac{5}{6}\right)$; HOLE

V.A. ; $x=2$, $x=3$; $m=1$ for (cross) both

H.A. ; $y=2$ → (from $\dfrac{2x^3}{x^3}=2$)

zeros @ $x=1$, $x=\dfrac{3}{2}$; $m=1$, $m=1$ (cross)



$R(0)=\dfrac{3}{6}=\dfrac{1}{2}$

zeros: $x-1=0$, $x=1$; $2x-3=0$, $2x=3$, $x=\dfrac{3}{2}$

9. (5 pts) Sketch the graph of $f(x)=\dfrac{(x+2)(x^2-1)}{(x+1)(x-3)}$. Show all intercepts, asymptotes and holes it has, if any.

$f(x)=\dfrac{(x+2)(x+1)(x-1)}{(x+1)(x-3)}=\dfrac{(x+2)(x-1)}{x-3}$, $x\neq -1$ $=\dfrac{x^2+x-2}{x-3}$, $x\neq -1$ $=f^*(x)$

$\mathcal{D}=\{x \mid x\neq -1 \text{ and } x\neq 3\}$

HOLE: $x=-1$

$f^*(-1)=\dfrac{(-1+2)(-1-1)}{(-1-3)}=\dfrac{(1)(-2)}{-4}=\dfrac{1}{2}$ ⇝ $(-1,\frac{1}{2})$ HOLE

V.A.: $x=3$

Zeros: $x=-2, 1$

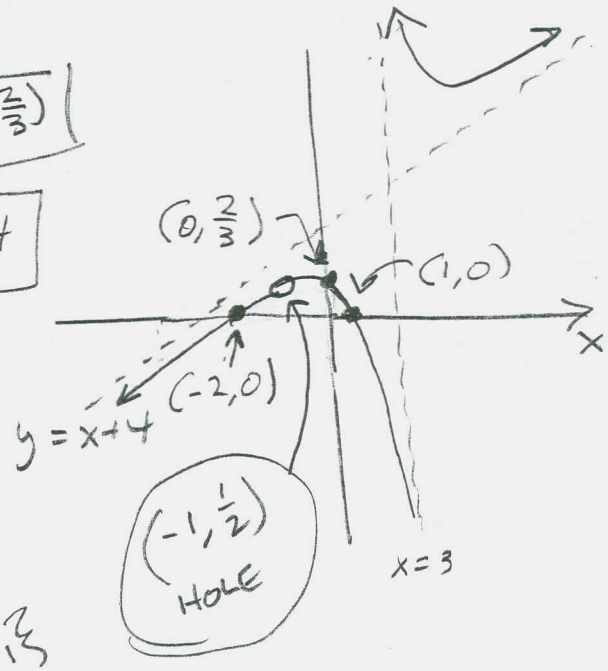
y-int: $(0,\frac{2}{3})$

O.A.: $3 \mid 1 \quad 1 \quad -2$

$\quad\quad\quad 3 \quad 12$

$\quad 1 \quad 4 \quad 10$

$y=x+4$ O.A.



Bonus (5 pts) What is the domain of $\sqrt{\dfrac{(x-1)^2}{(x+4)(x-5)^3}}$?

(Hint: Your previous work on this test may be of assistance.)

Same answer as #6b, only $x\neq -4$;

$(-\infty,-4)\cup(5,\infty)$